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(71) Applicant: TRAJECTA, INC. [US/US]; 611 South Congress Avenue, Suite 420, Austin, TX 78704 (US).

(72) Inventors: CARUANA, Frank, D.; 511 Oakwood Lane, Dripping Springs, TX 78620 (US). BEN-OR, Pinchas; 580 West 215, Manhattan, NY 10034 (US). CHATTERJEE, Abhijit; 203 West 39th Street, #201, Austin, TX 78751 (US). SMITH, Timothy, L.; 3418 Rain Forest Drive, Austin, TX 78746 (US). TRAUGHBER, Thomas, J.; 6104 Cape Coral Drive, Austin, TX 78746 (US). ALEXANDER, Rhonda; 5609 Painted Valley Drive, Austin, TX 78759 (US). NIEMANN, Michael, E.; 11702 Spotted Horse, Austin, TX 78759 (US). HARRIS, Matthew, M.; 1502 Alta Vista Avenue, Austin, TX 78704 (US). WALDSCHMIDT, Steven, J.; 2905 Etna Drive, Austin, TX 78733 (US).

(74) Agent: Hood, Jeffrey, C.; Conley, Rose & Tayon, P. C., P.O. Box 398, Austin, TX 78767-0398 (US).

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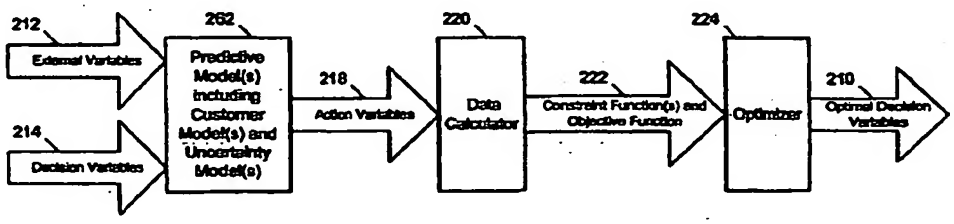
(54) Title: SYSTEM AND METHOD FOR ROBUST OPTIMIZATION INCLUDING UNCERTAINTY MODELS



(57) Abstract:

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TITLE: SYSTEM AND METHOD FOR ROBUST OPTIMIZATION INCLUDING UNCERTAINTY MODELS

BACKGROUND OF THE INVENTION

1. Field of the Invention

The present invention relates to the field of optimization, particularly the application of optimization science in the realm of strategic decision-making.

2. Description of the Related Art

Numerous approaches have been applied to the issue of strategic business decision-making, including decision support, data mining, and optimization.

Decision support typically utilizes technologies such as On-Line Analytical Processing (OLAP) and Relational On-Line Analytical Processing (ROLAP) to more systemically examine what has happened in the past. With these approaches, a querying process may take place whereby a decision-maker may, for example, analyze all available data associated with transactions and customers, or a marketing manager may query a database to ascertain how well an offer performed with a customer segment over a period of time. These tools can be thought of as reporting applications. The decision-maker can create custom reports and can be alerted if a query result falls out of a certain range. Performance can be measured and managed using these tools. Although it may be useful to look to the past before making decisions about the future, these tools are often inadequate to help the decision-maker look into the future to the see the consequences of the numerous actions at his or her disposal.

Another approach, data mining, may enable the decision-maker to look forward. Utilizing technologies such as neural networks and decision trees, data mining tools may be used to analyze large amounts of data and uncover patterns that lead to predictions of what future behavior a customer is likely to exhibit. The best tools in this category may be capable of identifying whom marketers should target based on predicted responses. However, the proactive prescription of what to do to meet business goals is still beyond their capabilities. It is still up to the manager to determine what action to take.

These aforementioned approaches may help decision-makers gain a better understanding of their customers, but it is still the manager who must use his or her experience and intuition to make the final decision of what to do. If managers are not careful, these tools may only push them into making incorrect and costly decisions. These tools may not use rigorous scientific approaches to determine at what point something should be done and, more importantly, what should not be done. Organizations relying solely on these types of inadequate tools may simply become more efficient at executing status quo decision-making strategies that have historically been followed.

Another approach, optimization, may address these problems at least in part. Optimization may combine intuition and intelligence in solving strategic decision-making problems. Unlike data mining, which starts with inputs and predicts an output, optimization may start with the output and seek the best actions to take to change the output to the company's advantage. Data mining may tend to predict the future, but with optimization, the decision-maker may take actions to change the future to the company's benefit. In many business environments, the goal of

optimization is to enable business decision-makers to determine which combinations and levels of treatments under their control should be applied. Typically, optimization solves a defined objective function, subject to one or more constraints, to select optimal values for one or more decision variables. Decision variables are the parameters over which the decision-maker has control. For instance, in many business applications, net profit is to be maximized or, if the decisions being modeled do not impact revenues, total cost minimized.

Typically, constraints are applied at the individual record level. These constraints may be referred to as local, or record-level, constraints. Cross-record constraints, on the other hand, may balance some resource or measurement across the entire set of records in the optimization simultaneously. Cross-record constraints limit the allocation of a global resource over all the records, such as budgets, total cost, etc. Applying cross-record constraints to an optimization problem tends to be a much more difficult task than applying local constraints, and so various heuristic techniques, such as the placement of Lagrangian multipliers in the objective function, have been used. However, these approaches are inexact and only approximate the impact of cross-record constraints. These techniques may also encounter significant problems if the number of cross-record constraints grows. Exact techniques of applying cross-record constraints are not believed to have been applied to customer relationship management or to the marketing of products to customers.

Another approach that may be brought to bear in the optimization process is robust optimization. This technique may address the issue of uncertainty management by solving for the optimum solution over a variety of possible futures over the time frame of interest. The uncertainties dealt with may originate from a number of sources, including environmental factors, such as the large-scale business climate, model errors, and input data gaps or errors. The goal of robust optimization is to find an optimal or near-optimal solution, which is not overly sensitive to any specific realization of the uncertainty of the problem. It is believed that the business application of this technique has been primarily in the area of financial investment portfolio management and not to customer relationship management or the marketing of products to customers.

Furthermore, optimization may be hampered by forecasting that ignores long-term considerations. In other words, current approaches may not adequately reflect behaviors that do not completely, directly, and empirically manifest themselves, for example, in a 6 or 12-month period.

For at least the foregoing reasons, there is a need for an improved system and method for optimization which addresses the problems discussed above and which may be applied to the area of customer relationship management.

SUMMARY OF THE INVENTION

The present invention provides various embodiments of a method and system for optimization of customer relationship management. Optimization may be used to select an optimal course of action for marketing one or more products to a plurality of customers. Customers may include current customers or prospective customers of a business. For example, customers may include credit customers to whom credit services are marketed, or customers may include pharmaceutical customers to whom pharmaceutical products are marketed. The one or more products may be marketed to customers in accordance with the optimal course of action, such as through direct mailing and/or targeted advertising. Other means of applying the optimal course of action may include, for example, conducting a re-pricing campaign in accordance with the optimal course of action, conducting an acquisition

campaign in accordance with the optimal course of action, conducting an e-mailing campaign in accordance with the optimal course of action, and conducting a promotional campaign for customer retention in accordance with the optimal course of action. As will be apparent to one skilled in the art, the system and method for optimization described herein may be applied to a wide variety of industries and circumstances.

An optimization process may accept the following elements as input: customer information records, predictive model(s) such as customer model(s), one or more constraints, and an objective. The optimization process may produce as output an optimized set of decision variables.

Customers may include individual customers, segments of like customers, and/or samples of customers. The customer information may include decision variables and external variables. Decision variables are those variables that the decision-maker may change to affect the outcome of the optimization process. For example, in the optimization of a credit card offer conducted by a credit card issuer, Annual Percentage Rate (APR) and credit limit may be decision variables. External variables are those variables that are not under the control of the decision-maker. For example, external variables may include variables such as customer addresses, customer income levels, customer demographic information, bureau data, transaction file data, cost of funds and capital, and other suitable variables.

In one embodiment, the customer information including decision variables and external variables may be input into the predictive model(s) to generate the action variables. In one embodiment, the predictive model(s) may be implemented as a neural network. The neural network may be trained, for example, with historical customer data records as input. In one embodiment, each of the predictive model(s) may correspond to one of the customer information records. Action variables are those variables that predict a set of actions for an input set of decision and external variables. In other words, the action variables may comprise predictive metrics for customer behavior. For example, in the optimization of a product marketing campaign, the action variables may include the probability of a customer's response to an offer. In the optimization of a credit card offer, the action variables may include predictions of balance, attrition, charge-off, purchases, payments, and other suitable behaviors for the customer of a credit card issuer.

In one embodiment, the objective function may be a function of lifetime customer value (LCV). The particular function which expresses lifetime customer value may vary from optimization process to optimization process. Generally, LCV may include a sum of discounted cash flows over the lifetime of one or more customer relationships. The combination of the long-term view and short-term view may be implemented by the use of both constraint(s) and objective to embody the LCV. For example, one or more constraints may be used to ensure that the business maintains a particular level of revenue over the short term, while the objective may be used to ensure that the business maximizes the value of the customer relationships over the long term. Likewise, the objective may include short-term considerations and the constraints may include long-term considerations.

Constraints are typically "real-world" limits on the decision variables. In one embodiment, the constraints include cross-record constraints. Cross-record constraints are limitations on one or more global resources as applied to a plurality of records. For example, a cross-record constraint may define a limitation on a quarterly budget or other finite resource that the optimization process is expected to meet. The constraints may also include one or more record-level constraints. A record-level constraint is a limitation that is applied to one record at a time.

For example, a record-level constraint may be a limitation on an interest rate offered to a customer by a credit card issuer. In one embodiment, the optimization process may use a cross-record objective.

In one embodiment, the optimization process may employ robust optimization. Solving the objective function for a robust optimization process may include solving the objective function across a plurality of uncertainty models to select the optimal solution. An uncertainty model may include any model which adds a range of uncertainty to an optimization process. Uncertainty models may be generated by sampling a probability distribution. In one embodiment, the uncertainty models may include scenarios. As used herein, "scenarios" include multiple sets of possible values for uncertain data or models. For example, a robust optimization process may use scenarios to determine an optimal solution across a range of possible models of a customer in the future. The scenarios may be designed so as to span the range of possibilities likely to occur over the time frame of interest.

In one embodiment, a method for optimization of customer relationship management may include the following steps. A set of customer data and/or other data corresponding to one or more customers may be collected and received. Given the set of customer data, one or more decision variables may be optimized based on an objective and one or more constraints to produce optimized decision variables. The one or more terms of a product may be set to the optimized decision variables to produce an optimized product. The optimized product may comprise a set of optimized terms. For example, if product is a credit card, then the optimized terms may include an optimal interest rate and credit limit. Generally, setting the decision variables of the product to the optimized decision variables may result in increasing the value of the product vendor's relationships with customers of the product.

The optimized product may be offered to one or more of the customers. In various embodiments, the optimized product may be offered to current and/or prospective customers in a variety of ways and under a variety of circumstances. For example, the optimized product may be offered via a direct mailing, targeted telephone advertising, a re-pricing campaign, e-mail, and/or a promotional campaign for customer retention. In appropriate cases, a response from the customer to the offering may be accepted. In various embodiments, the response may be accepted automatically and without human intervention, such as through an automated on-line or web-based system, or the response may be accepted with human intervention. In some cases, such as changing the APR of a credit card, for example, receiving a customer response may not be pertinent.

The optimized terms may be carried out during a lifetime of the optimized product. In other words, the product vendor may maintain the customer relationship for a particular product according to the optimized terms produced in the optimization process. Carrying out the optimized terms may include conducting one or more transactions according to the optimized terms during the product lifetime. For example, a credit card issuer may honor the interest rate and credit limit of the optimized offer for one or more credit-card transactions for as long as the customer holds the credit card.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is an illustration of a typical computer system which is suitable for implementing various embodiments;

Figure 2 is a network diagram of an illustrative distributed computing environment which is suitable for implementing various embodiments;

Figure 3a is a block diagram which illustrates an overview of optimization according to one embodiment;

Figure 3b is a dataflow diagram which illustrates an overview of optimization according to one embodiment;

Figure 4 illustrates a single model according to one embodiment;

Figure 5 illustrates multiple models for multiple products and a single customer according to one embodiment;

Figure 6 illustrates multiple models for multiple customers and a single product according to one embodiment;

Figure 7a is a block diagram which illustrates an optimization process with cross-record constraints or a cross-record objective according to one embodiment;

Figure 7b is a dataflow diagram which illustrates an optimization process with cross-record constraints or a cross-record objective according to one embodiment;

Figures 8a through 8e illustrate examples of probability distributions associated with a robust optimization process according to one embodiment;

Figure 9a is a block diagram which illustrates a robust optimization process according to one embodiment;

Figure 9b is a dataflow diagram which illustrates a robust optimization process according to one embodiment;

Figure 10 illustrates multiple scenario models for product-customer pairs according to one embodiment;

Figure 11a is a block diagram which illustrates an optimization process with an objective function of lifetime customer value (LCV) according to one embodiment;

Figure 11b is a dataflow diagram which illustrates an optimization process with an objective function of lifetime customer value (LCV) according to one embodiment;

Figure 12a is a block diagram which illustrates a robust optimization process with cross-record constraints/objective and an objective function of lifetime customer value (LCV) according to one embodiment;

Figure 12b is a dataflow diagram which illustrates a robust optimization process with cross-record constraints/objective and an objective function of lifetime customer value (LCV) according to one embodiment;

Figure 13 is a flowchart which illustrates optimization of customer relationship management according to one embodiment;

Figure 14 illustrates an off-line system architecture for optimization of customer relationship management according to one embodiment;

Figure 15 illustrates an import module for optimization of customer relationship management according to one embodiment;

Figure 16 illustrates an export module for optimization of customer relationship management according to one embodiment;

Figure 17 illustrates an design experiment module for optimization of customer relationship management according to one embodiment;

Figure 18 illustrates an experiment engine module for optimization of customer relationship management according to one embodiment;

Figure 19 illustrates a build components module for optimization of customer relationship management according to one embodiment;

Figure 20 illustrates an assemble/setup components module for optimization of customer relationship management according to one embodiment;

Figure 21 illustrates a manage questions sub-module for optimization of customer relationship management according to one embodiment;

Figure 22 illustrates a manage optimization sub-module for optimization of customer relationship management according to one embodiment;

Figure 23 illustrates a manage marginal optimization sub-module for optimization of customer relationship management according to one embodiment;

Figure 24 illustrates a decision engine module for optimization of customer relationship management according to one embodiment;

Figure 25 illustrates a decision views module for optimization of customer relationship management according to one embodiment;

Figure 26 illustrates a decision reports module for optimization of customer relationship management according to one embodiment;

Figure 27 illustrates an on-line experiment architecture for optimization of customer relationship management according to one embodiment;

Figure 28 illustrates an on-line decision architecture for optimization of customer relationship management according to one embodiment.

While the invention is susceptible to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and will herein be described in detail. It should be understood, however, that the drawings and detailed description thereto are not intended to limit the invention to the particular form disclosed, but on the contrary, the intention is to cover all modifications, equivalents, and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

DETAILED DESCRIPTION OF SEVERAL EMBODIMENTS

Figure 1: A Typical Computer System

Figure 1 illustrates a typical computer system 150 which is suitable for implementing various embodiments. Each computer system 150 typically includes components such as a CPU 152 with an associated memory medium, represented by floppy disks 160. The memory medium may store program instructions for computer programs, wherein the program instructions are executable by the CPU 152. The computer system 150 may further include a display device such as a monitor 154, an alphanumeric input device such as a keyboard 156, and a directional input device such as a mouse 158. The computer system 150 is operable to execute the computer programs to implement an improved optimization system and method as described herein.

The computer system 150 preferably includes a memory medium on which computer programs according to various embodiments may be stored. The term "memory medium may include an installation medium, e.g., a CD-ROM, or floppy disks 160, a computer system memory such as DRAM, SRAM, EDO RAM, Rambus RAM, etc., or a

non-volatile memory such as a magnetic media, e.g., a hard drive, or optical storage. The memory medium may include other types of memory as well, or combinations thereof. In addition, the memory medium may be located in a first computer in which the programs are executed, or may be located in a second different computer which connects to the first computer over a network. In the latter instance, the second computer provides the program instructions to the first computer for execution. Also, the computer system 150 may take various forms, including a personal computer system, mainframe computer system, workstation, network appliance, Internet appliance, personal digital assistant (PDA), television system or other device. In general, the term "computer system" can be broadly defined to encompass any device having a processor which executes instructions from a memory medium.

The memory medium preferably stores a software program or programs for event-triggered transaction processing as described herein. The software program(s) may be implemented in any of various ways, including procedure-based techniques, component-based techniques, and/or object-oriented techniques, among others. For example, the software program may be implemented using ActiveX controls, C++ objects, JavaBeans, Microsoft Foundation Classes (MFC), or other technologies or methodologies, as desired. A CPU, such as the host CPU 152, executing code and data from the memory medium includes a means for creating and executing the software program or programs according to the methods and/or block diagrams described below.

The computer system for optimization of customer relationship management as discussed herein may be a typical computer system 150 as illustrated in Figure 1.

Figure 2: A Distributed Computing Environment

Figure 2 illustrates a distributed or enterprise computing environment according to one embodiment. A distributed computer system or enterprise 100 includes a plurality of computer systems which are interconnected through one or more networks. Although one particular embodiment is shown in Figure 2, the distributed computer system 100 may include a variety of heterogeneous computer systems and networks which are interconnected in a variety of ways and which run a variety of software applications and/or operating system software.

One or more local area networks (LANs) 104 may be included in the enterprise 100. A LAN 104 is a network that spans a relatively small area. Typically, a LAN 104 is confined to a single building or group of buildings. Each node (i.e., individual computer system or device) on a LAN 104 preferably has its own CPU with which it executes programs, and each node is also able to access data and devices anywhere on the LAN 104. The LAN 104 thus allows many users to share devices (e.g., printers) as well as data stored on file servers. The LAN 104 may be characterized by any of a variety of types of topology (i.e., the geometric arrangement of devices on the network), of protocols (i.e., the rules and encoding specifications for sending data, and whether the network uses a peer-to-peer or client/server architecture), and of media (e.g., twisted-pair wire, coaxial cables, fiber optic cables, radio waves). As illustrated in Figure 2, the distributed computer system 100 may include one LAN 104. However, in alternate configurations the distributed computer system 100 may include a plurality of LANs 104 which are coupled to one another through a wide area network (WAN) 102. A WAN 102 is a network that spans a relatively large geographical area.

Each LAN 104 includes a plurality of interconnected computer systems and optionally one or more other devices: for example, one or more workstations 110a, one or more personal computers 112a, one or more laptop or notebook computer systems 114, one or more server computer systems 116, and one or more network printers 118.

As illustrated in Figure 2, an example LAN 104 may include one of each of computer systems 110a, 112a, 114, and 116, and one printer 118. The LAN 104 may be coupled to other computer systems and/or other devices and/or other LANs 104 through a WAN 102.

One or more mainframe computer systems 120 may be coupled to the distributed computer system 100. As shown in Figure 2, the mainframe 120 may be coupled to the distributed computer system 100 through the WAN 102, but alternatively one or more mainframes 120 may be coupled to the distributed computer system 100 through one or more LANs 104. As shown, the mainframe 120 may be coupled to a storage device or file server 124 and mainframe terminals 122a, 122b, and 122c. The mainframe terminals 122a, 122b, and 122c may access data stored in the storage device or file server 124 coupled to or included in the mainframe computer system 120.

The distributed computer system 100 may also include one or more computer systems which are connected to the distributed computer system 100 through the WAN 102: as illustrated, a workstation 110b and a personal computer 112b. In other words, the enterprise 100 may optionally include one or more computer systems which are not coupled to the distributed computer system 100 through a LAN 104. For example, the distributed computer system 100 may include computer systems which are geographically remote and connected to the distributed computer system 100 through the Internet.

Figures 3 through 6: Overview of Optimization

As discussed herein, optimization may generally be used by a decision-maker associated with a business to select an optimal course of action or optimal course of decision. The optimal course of action or decision may include a sequence or combination or actions and/or decisions. For example, optimization may be used to select an optimal course of action for marketing one or more products to one or more customers. As used herein, a "customer" may include an existing customer or a prospective customer of the business. As used herein, a "customer" may include one or more persons, one or more organizations, or one or more business entities. As used herein, the term "product" is intended to include various types of goods or services, such as books, music, content subscription services, furniture, online auction items, clothing, ISP service, consumer electronics, travel, software, pharmaceutical or medical supplies, computer systems, etc., or various services such as loans (e.g., credit card, auto, mortgage, and home re-financing loans), securities (e.g., CDs, retirement accounts, cash management accounts, and mutual funds), or insurance (e.g., life, health, auto, and home owner's insurance), among others. For example, customers may include credit customers to whom credit services are marketed, or customers may include pharmaceutical customers to whom pharmaceutical products are marketed. As will be apparent to one skilled in the art, the system and method for optimization described herein may be applied to a wide variety of industries and circumstances.

Generally, a business may desire to apply the optimal course of action or optimal course of decision to one or more customer relationships to increase the value of customer relationships to the business. As used herein, a "portfolio" includes a set of relationships between the business and a plurality of customers. In general, the process of optimization may include determining which variables in a particular problem are most predictive of a desired outcome, and what treatments, actions, or mix of variables under the decision-maker's control (i.e., decision variables) will optimize the specified value. The one or more products may be marketed to customers in accordance with the optimal course of action, such as through direct mailing and/or targeted advertising. Other means of applying the optimal course of action may include, for example, conducting a re-pricing campaign in accordance

with the optimal course of action, conducting an acquisition campaign in accordance with the optimal course of action, conducting an e-mailing campaign in accordance with the optimal course of action, and conducting a promotional campaign in accordance with the optimal course of action.

Figure 3a is a block diagram which illustrates an overview of optimization according to one embodiment. Figure 3b is a dataflow diagram which illustrates an overview of optimization according to one embodiment. As shown in Figure 3a, an optimization process 200 may accept the following elements as input: customer information records 202, predictive model(s) such as customer model(s) 204, one or more constraints 206, and an objective 208. The optimization process 200 may produce as output an optimized set of decision variables 210. In one embodiment, each of the customer model(s) 204 may correspond to one of the customer information records 202. As used herein, an "objective" may include a goal or desired outcome of an optimization process.

As used herein, a "constraint" may include a limitation on the outcome of an optimization process. Constraints are typically "real-world" limits on the decision variables and are often critical to the feasibility of any optimization solution. Managers who control resources and capital or are responsible for financial effects should be involved in setting constraints that accurately represent their real-world environments. Setting constraints with management input may realistically restrict the allowable values for the decision variables.

In many applications of the optimization process 200, the number of customers involved in the optimization process 200 may be so large that treating the customers individually is computationally unfeasible. In these cases, it may be useful to group like customers together in segments. If segmented properly, the customers belonging to a given segment will typically have approximately the same response in the action variables to a given change in decision variables and external variables. For example, customers may be placed into particular segments based on particular customer attributes such as risk level, financial status, or other demographic information. Each customer segment may be thought of as an average customer for a particular type or profile. A segment model, which represents a segment of customers, may be used as described above with reference to a customer model 204 to generate the action variables for that segment. Another alternative to treating customers individually is to sample a larger pool of customers. Therefore, as used herein, a "customer" may include an individual customer, a segment of like customers, and/or a sample of customers. As used herein, a "customer model", "predictive model", or "model" may include segment models, models for individual customers, and/or models used with samples of customers.

The customer information 202 may include decision variables 214 and external variables 212. As used herein, "decision variables" are those variables that the decision-maker may change to affect the outcome of the optimization process 200. For example, in the optimization of a credit card offer conducted by a credit card issuer, Annual Percentage Rate (APR) and credit limit may be decision variables. As used herein, "external variables" are those variables that are not under the control of the decision-maker. In other words, the external variables are not changed in the decision process but rather are taken as givens. For example, external variables may include variables such as customer addresses, customer income levels, customer demographic information, bureau data, transaction file data, cost of funds and capital, and other suitable variables.

In one embodiment, the customer information 202 including decision variables 214 and external variables 212 may be input into the predictive model(s) 216 to generate the action variables 218. In one embodiment, each of the predictive model(s) 216 may correspond to one of the customer information records 202, wherein each of the

customer information records 202 may include appropriate decision variables 214 and external variables 212. As used herein, "action variables" are those variables that predict a set of actions for an input set of decision and external variables. In other words, the action variables may comprise predictive metrics for customer behavior. For example, in the optimization of a product marketing campaign, the action variables may include the probability of a customer's response to an offer. In a re-pricing campaign, the action variables may include the likelihood of a customer maintaining a service after re-pricing the service. In the optimization of a credit card offer, the action variables may include predictions of balance, attrition, charge-off, purchases, payments, and other suitable behaviors for the customer of a credit card issuer.

The predictive model(s) 216 may include the customer model(s) 204 as well as other models. The predictive model(s) 216 may take any of several forms, including, but not limited to: trained neural nets, statistical models, analytic models, and any other suitable models for generating predictive metrics. The models may take various forms including linear or non-linear, such as a neural network, and may be derived from empirical data or from managerial judgment.

In one embodiment, the predictive model(s) 216 may be implemented as a neural network. Typically, the neural network may include a layer of input nodes, interconnected to a layer of hidden nodes, which are in turn interconnected to a layer of output nodes, wherein each connection is associated with an adjustable weight whose value is set in the training phase of the model. The neural network may be trained, for example, with historical customer data records as input. The trained network may include a non-linear mapping function that may be used to model customer behaviors and provide predictive customer models in the optimization system. The trained neural network may generate action variables 218 based on customer information 202 such as external variables 212 and decision variables 214.

In one embodiment, a model comprises a representation that allows prediction of action variables, a, due to various decision variables, d, and external variables, e. Figure 4 illustrates a model 215 with external variables 212, decision variables 214, and resulting action variables 218. For example, a customer may be modeled to predict customer response to various offers under various circumstances. It may be said that the action variables, a, are a function, via the model, of the decision and external variables, d and e, such that:

$$a = M(d,e) \tag{1}$$

wherein $M()$ is the model, a, is the vector of action variables, d is the vector of decision variables, and e is the vector of external variables.

In one embodiment, the action variables 218 generated by the model(s) 216 may be used to formulate constraint(s) 206 and the objective function 208 via formulas. In Figure 3b, a data calculator 220 generates the constraint(s) and objective 222 using the action variables 218 and potentially other data and variables. In one embodiment, the formulas used to formulate the constraint(s) and objective 222 may include financial formulas such as formulas for determining net operating income over a certain time period. The constraint(s) and objective 222 may be input into an optimizer 224, which may comprise, for example, a custom-designed process or a commercially available "off the shelf" product. The optimizer may then generate the optimal decision variables 210 which have values optimized for the goal specified by the objective function 208 and subject to the constraint(s)

206. The optimization process 200 carried out by the optimizer 224 is discussed in greater detail below. A further understanding of the optimization process 200 may be gained from the references "An Introduction to Management Science: Quantitative Approaches to Decision Making", by David R. Anderson, Dennis J. Sweeney, and Thomas A. Williams, West Publishing Co. (1991); and "Fundamentals of Management Science" by Efraim Turban and Jack R. Meredith, Business Publications, Inc. (1988).

Overview of Optimization for a Single Customer

Many optimization problems have the following form: given a model of a customer or segment $a = M(d,e)$, a set of objective parameters o, a set of constraint parameters c_p, and a set of constraint bounds c_b, use an optimizer to compute the set of decision variables for a customer or segment that extremizes (e.g., maximizes or minimizes) an objective function of the form:

$$J = f(d,e,a,o) \tag{2}$$

subject to the model constraint:

$$a = M(d,e) \tag{1}$$

and a general set of constraints of the form:

$$0 \le g(d,e,a,c_p) \le c_b \tag{3}$$

wherein the decision variables d, are a subset of the set of possible decision variables, D.

There are a number of approaches for solving optimization problems of this form. As is well known by those skilled in the art, the approach selected depends on the form of the model, of the objective function, of the constraints, and of the set of possible decision variables. The model, objective function, and constraints may each be either linear (L) or non-linear (NL). The decision variable set, D, may be a linearly bounded single region (simple convex area) (L), a non-linear bounded single region (NL), multiple regions (MR), or discrete. Commercial solvers, or optimizers, are available for solving all combinations of linear and non-linear components for single region decision variable sets. For the cases when variables are not restricted to a single continuous region, a variety of other, more heuristic, approaches are generally available. Several of these approaches solving optimization problems are discussed in greater detail as follows.

Example of Optimization with Linear Programming

A simple credit card offer optimization problem illustrates the LP approach. The model computes the response rate to a mailed offer and expected monthly balance of a responder; therefore, the action variables are: a_1

= response rate; a_2 = expected balance. The decision variables for the offer are annual percentage rate (APR) and credit limit; thus, d_1 = APR; d_2 = credit limit. There are no external variables in this example.

A linear model of the form:

$$a_1 = w_{11}d_1 + w_{12}d_2 + b_1 \qquad (4)$$
$$a_2 = w_{21}d_1 + w_{22}d_2 + b_2$$

may be used, wherein w_{11}, w_{12}, b_1, w_{21}, w_{22}, and b_2 are parameters of the model. The parameters may be found, for example, using linear regression techniques based upon historical data.

The objective function, J, to be maximized, also linear, is of the form:

$$J = o_1 a_1 + a_2 \qquad (5)$$

wherein o_1 is an optimization parameter. Using this objective function, a_1 and a_2 may be maximized. The relative importance of a_1 versus a_2 is determined by the optimization parameter, o_1, which is specified by the user.

A linear constraint is of the form:

$$0 \le c_{p,1}a_1 + a_2 \le c_{b,1} \qquad (6)$$

and the set of possible decision variables, D, is restricted such that:

$$.05 \le d_1 \le .19 \quad \text{(range of APR)} \qquad (7)$$
$$1{,}000 \le d_2 \le 5{,}000 \quad \text{(range of credit limit)} \qquad (8)$$

Because the model, objective function, constraint, and set of decision variables are linear, this example can be solved using standard linear programming techniques.

Example of Optimization with Non-Linear Programming

In a further example, the objective, constraints, and set of decision variables are of the form shown above, and the model is implemented by a non-linear neural network:

$$a = M(d,e) = NN(d,e,w) \qquad (9)$$

wherein w is the vector of weight parameters of the neural network which may be identified using historical data and the back propagation method of neural network training. In this case, because the model is non-linear. a non-linear commercial solver/optimizer may be used to solve for the decision variables.

Overview of Optimization with Heuristic Linear Programming

A similar problem may be considered, but with the set of possible decision variables restricted to a discrete set, such as:

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \in \left(\begin{bmatrix} .05 \\ 1000 \end{bmatrix}, \begin{bmatrix} .09 \\ 1000 \end{bmatrix}, \begin{bmatrix} .125 \\ 2500 \end{bmatrix}, \begin{bmatrix} .19 \\ 5000 \end{bmatrix} \right) \tag{10}$$

such that the APR values of each of the four offers are 5%, 9%, 12.5%, and 19%, respectively, and the credit limits are $1000, $1000, $2500, and $5000, respectively. Because the problem is non-linear and discrete, a mixed integer linear programming (MILP) approach may be used; however, by reformulating the problem heuristically, a linear programming (LP) technique may be used instead. This is referred to as a heuristic LP approach.

The above problem may be reformulated by enumerating the solutions, e.g., by computing the output of the neural network model for each element of the set:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \in (NN(.05,1000), NN(.09,1000), NN(.125,2500), NN(.19,5000)) = \tag{11}$$

$$\left(\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}, \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}, \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}, \begin{bmatrix} a_{14} \\ a_{24} \end{bmatrix} \right)$$

wherein the output corresponding to the first element of the set is $\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$. The objective function, J, may then be rewritten to select the optimal pair of action variables:

$$\max J = n_1(o_1 a_{11} + a_{21}) + n_2(o_2 a_{12} + a_{22}) + n_3(o_3 a_{13} + a_{23}) + n_4(o_4 a_{14} + a_{24}) \tag{12}$$

wherein n_i are selection variables constrained by:

$$0 \le n_i \le 1 \tag{13}$$

(14)

$$\sum_{i=1}^{k} n_i = 1$$

and wherein:

$$0 \le c_{p,1} a_1 + a_2 \le c_{b,1} \tag{6}$$

At this point, a conventional linear programming technique may be used because the selection variables n_i are optimized, rather than the decision variables, and because n_i appear linearly. Once the optimal selection variables are computed, one of the n_i will be equal to 1, with the rest equal to 0, assuming only one maximum in the set. The optimal decision variables of the set correspond to the n_i equal to 1. Thus, the decision variables are computed using a heuristic LP approach. This technique may generally be used when the set of discrete decision variables is finite.

Overview of Optimization for Multiple Products or Multiple Customers

In the cases where multiple products may be offered to a single customer, or where a single product may be offered to multiple customers, a different model may be used for each product/customer pair. For example, in the case of a single customer being offered multiple products, the models may be defined as follows:

$$a_1 = M(d_1, e_1),\ a_2 = M(d_2, e_2), \ldots;\ a_m = M(d_m, e_m) \tag{15}$$

with a set of Boolean selection variables:

$$s = (s_1, s_2, \ldots s_m) \tag{16}$$

that may be used to select which offers to make to the customer. Figure 5 illustrates the multiple product models of expression (15).

In a similar way, for the case of a single product being offered to multiple customers, the models may be defined as follows:

$$a_1 = M(d_1, e_1),\ a_2 = M(d_2, e_2), \ldots,\ a_N = M(d_N, e_N) \tag{17}$$

with a set of Boolean selection variables:

$$s = (s_1, s_2, \ldots s_N) \tag{18}$$

Figure 6 illustrates these multiple customer models. Optimization with multiple customer models is mathematically equivalent to optimization with multiple product models.

Given the above set of models for various products $M \in (M_1, M_2, \ldots, M_m)$ for a customer (i.e., an individual customer, customer segment, or customer sample), a set of objective parameters o, a set of constraint parameters c_p, a set of constraint bounds c_b, and an objective function of the form:

$$J = f(d_1, e_1, a_1, s_1, d_2, e_2, a_2, s_2, \ldots d_m, e_m, a_m, s_m, o) \tag{19}$$

which is subject to the model constraints:

$$a_1 = M(d_1, e_1),\ a_2 = M(d_2, e_2), \ldots,\ a_m = M(d_m, e_m) \tag{15}$$

and a general set of constraints of the form:

$$0 \le g(d_1, e_1, a_1, s_1, d_2, e_2, a_2, s_2, \ldots d_m, e_m, a_m, s_m, c_p) \le c_b \tag{20}$$

and the set of possible decision variables, D, and selection variables, S, such that:

$$d \in D$$
$$s \in S \tag{21}$$

then an optimizer may generally be used to compute the decision variables for a customer to extremize the objective function, J.

Figure 7: Optimization with Cross-Record Constraints

Figure 7a is a block diagram which illustrates an optimization process 200 with cross-record constraints according to one embodiment. Figure 7b is a dataflow diagram which illustrates an optimization process 200 with cross-record constraints according to one embodiment. In one embodiment, the constraint(s) 252 may include one or more cross-record constraint(s), and the optimizer 224 may solve the objective 208 subject to the cross-record constraint(s) 252. As used herein, a "cross-record constraint" is a limitation on one or more global resources as applied to a plurality of records such as customer information records 202. For example, a cross-record constraint may define a limitation on a quarterly budget or other finite resource that the optimization process 200 is expected

to meet. The constraint(s) 252 may also include one or more record-level constraints. As used herein, a "record-level constraint" is a limitation that is applied to one record at a time. For example, a record-level constraint may be a limitation on an interest rate offered to a customer by a credit card issuer.

The optimization process 200 may also be solved subject to limitations on global resources by using a cross-record objective function 209. Therefore, the data calculator 220 may generate cross-record constraint(s) and/or cross-record objective 254 for use by the optimizer 224. An objective function is a cross-record objective function if it cannot be written as a sum of individual record objectives:

$$J = f(d_1, e_1, a_1, s_1, d_2, e_2, a_2, s_2, \ldots d_m, e_m, a_m, s_m, o) \neq f_1(d_1, e_1, a_1, s_1, o) + f_2(d_2, e_2, a_2, s_2, o) + \ldots + f_m(d_m, e_m, a_m, s_m, o) \quad (22)$$

An example of such an objective might be the profit for a collection of product offers.

Figures 8 through 10: Robust Optimization

Figure 9a is a block diagram which illustrates a robust optimization process according to one embodiment. Figure 9b is a dataflow diagram which illustrates a robust optimization process according to one embodiment. Generally, robust optimization includes constrained optimization across a plurality of uncertainty models 260. In other words, solving the objective function in robust optimization may include solving the objective function across a plurality of uncertainty models 260 to select the optimal solution, given the uncertainty provided by the uncertainty models 260. As used herein, an "uncertainty model" may include any model which adds a range of uncertainty to an optimization process. "Robust optimization" is generally synonymous with stochastic or probabilistic robust optimization.

In a deterministic model, a change in decision variables or external variables results in exactly (i.e., deterministically) the same change in the resulting action variables. Thus, a deterministic model $f(.)$ may be expressed as follows:

$$a = f(d, e) \quad (23)$$

In a nondeterministic or uncertainty model, a change in decision variables or external variables may not result in exactly the same change in the resulting action variables. In this case, the action variables may be represented by a probability distribution:

$$p(a|d, e) \quad (24)$$

The model (24) is the probability distribution of the action variables, a, given the decision and external variables, d and e. In a preferred embodiment, robust optimization uses a nondeterministic or uncertainty model for a customer/segment, as shown in expression (24), rather than a deterministic model.

In many cases, the direct probability distribution cannot be used in the optimization problem, either because the distribution cannot be computed in a reasonable time or the optimization problem cannot be computed in a reasonable time. In these cases, samples of the distribution may be used to solve or approximate the solution to the problem. Sampling is illustrated by the following simple problem. The expected value of a distribution, $f(x)$, can be approximated by drawing N random samples from $f(x)$. Thus:

$$E(x) = \int x f(x) dx \cong \frac{1}{N} \sum_{i=1}^{N} x_i \tag{25}$$

wherein x_i are the independent samples of $f(x)$. The independent samples are known as scenarios. By independently sampling the probability distribution $p(a|d,e)$, a set of scenario models $\{a_1 = f_1(d,e), a_2 = f_2(d,e), \mathrm{K}\ , a_N = f_N(d,e)\}$ may be generated. The scenario models may then be used to approximate the robust optimization solution. The solution may be determined as shown above for multiple models.

Bayes' Rule allows us to express the probability distribution (24) as:

$$p(a|d,e) = p(d,e|a) p_N(a) \tag{26}$$

wherein $p_N(a)$ is known as the prior distribution and is normalized. By assuming a prior distribution, $\tilde{p}(a)$, the probability distribution may be approximated as:

$$p(a|d,e) \cong p(d,e|a) \tilde{p}(a) \tag{27}$$

In general, it is easier to sample $p(d,e|a)$ than $p(a|d,e)$. Thus, sampling of $p(d,e|a)$ may be used in many cases. Sampling models and optimization approaches using this technique are further discussed in a patent application entitled "Bayesian Neural Network for Optimization" by E. Hartman and C. Peterson, S/N 09/290,791, filed 4/12/99, which is incorporated herein by reference, as though fully and completely set forth herein.

A second approach to sampling the distribution, $p(a|d,e)$, is realized by:

$$p(a|d,e) = f_{AVE}(d,e) + p(r|d,e) \tag{28}$$

wherein $f_{AVE}(d,e)$ is a deterministic function representing the average of the distribution, $p(a|d,e)$. and wherein $p(r|d,e)$ is the residual distribution. In this case, the residual distribution is sampled rather than the actual distribution.

It is common to train a neural network to represent $f_{AVE}(d,e)$. Thus:

$$p(a|d,e) \equiv NN_{AVE}(d,e) + p(r|d,e) \tag{29}$$

Furthermore, it may be assumed that $p(r|d,e)$ is independent of *d*, *e*. Thus:

$$p(r|d,e) \equiv p(r) \tag{30}$$

and

$$p(a|d,e) \equiv NN_{AVE}(d,e) + p(r) \tag{31}$$

Given an historical dataset, *D*, the NN_{AVE} can be developed. The historical dataset can also be used to determine the distribution of the residuals. In this case, the distribution is dependent on the historical data, and thus:

$$p(r|D) \tag{32}$$

is the distribution. This results in the model:

$$p(a|d,e) \equiv NN_{AVE}(d,e) + p(r|D) \tag{33}$$

One form of the distribution for $p(r|D)$ is a normal distribution with zero mean $N(\sigma|D)$, as illustrated in Figure 8a, and wherein σ is the standard deviation. Given the residuals, for the historical dataset *D*, the standard deviation may be computed, giving the model:

$$p(a|d,e) \equiv NN_{AVE}(d,e) + N(\sigma|D) \tag{34}$$

It is straightforward to sample this distribution and create scenario models.

Another distribution is the symmetric triangle distribution $T(w)$ shown in Figure 8b, wherein w is the width. The residual data may be used to determine w for the distribution:

$$p(a|d,e) \cong NN_{AVE}(d,e) + T(w|D) \tag{35}$$

The distributions may be sampled, yielding models of the form:

$$\begin{aligned} a_1 &= NN_{AVE}(d,e) + r_1 \\ a_2 &= NN_{AVE}(d,e) + r_2 \\ &\text{M} \\ a_N &= NN_{AVE}(d,e) + r_N \end{aligned} \tag{36}$$

wherein $r_1, r_2, \ldots, r_N$ are samples of $p(r|D)$.

Because events of the future may differ from the past, the user may override the historical data and use his or her best judgment to select an appropriate distribution. Robust optimization may therefore be enhanced by management judgment. For example, consider the triangle distribution expressed as distribution (35), or by applying Bayes' Rule:

$$p(a|d,e) \cong NN_{AVE}(d,e) + T(D|w)T(w) \tag{37}$$

The user may select an appropriate prior distribution for $T(w)$. Often, a broad Gaussian distribution may be selected, as shown in Figure 8c. However, the user may feel that the world is too uncertain to trust the historical data as a guide to future events. Therefore, the user can select a prior distribution with a high value for w to represent a region of higher uncertainty, as shown in Figure 8d. The user may also select the distribution shown in Figure 8e, resulting in the following model:

$$p(a|d,e) \cong NN_{AVE}(d,e) + T(W) \tag{38}$$

By setting the prior distribution in this way, management judgment can be taken into account in the probability models and thus in robust optimization.

In one embodiment, the uncertainty models 260 may include scenarios as discussed above. As used herein, "scenarios" include multiple sets of possible values for uncertain data. Scenarios may express a range of uncertainty in the future. The scenarios may be designed so as to span the range of possibilities likely to occur over the time frame of interest. In this way, the decision-maker may measure the consequences of environmental uncertainty, model errors, gaps or errors in input data, or errors experienced in predicted and forecasted behaviors

or other business parameters that impact his or her decision metric, and in doing so, determine his or her strategy based upon the desired level of risk.

As shown in Figure 10, multiple scenarios may be represented by multiple models for a product-customer/segment pair:

$$a_1 = M(d_1, e_1),\ a_2 = M(d_2, e_2), \ldots,\ a_R = M(d_R, e_R) \tag{39}$$

The notation is similar to that for multiple products/customers as discussed with reference to Figures 5 and 6, except that instead of Boolean selection variables, weights may be used to weight the importance or likelihood of one model against another. A probability may be assigned to each scenario to denote its likelihood of occurrence. These probabilities may be used to weight the contributions of each record's constraint and objective calculations, therefore allowing the decision-maker to assess the consequences of strategic decisions in the light of the relative associated risks.

In one embodiment, the plurality of uncertainty models 260 may be generated by varying a set of action variables 218 to represent varying circumstances of possible futures. Uncertainty models may be used to vary the values of the action variables 218 according to (1) some set of empirical distributions (one or more distributions per action variable) or to (2) some set of distributions derived through managerial judgment or to (3) some combination of empirically derived and judgmentally derived distributions. The objective function may be a function of the action variables 218 across a plurality of uncertainty models 260.

In another embodiment, probabilities are not assigned to the scenarios; rather, a decision table may be used in conjunction with any of a number of well-established methods, such as Equally Likely Criterion, Maximax Criterion, Maximin Criterion, Hurwicz Criterion of Realism, or Minimax Regret, as will be familiar to those skilled in the art. A greater understanding of these established methods may be gained by referring to an elementary text on Management Science or Decision Analysis such as "Spreadsheet Modeling and Decision Analysis: A Practical Introduction to Management Science", Cliff T. Ragsdale, Course Technology Inc. (1997).

Figure 11: Optimization of Lifetime Customer Value

In the past, businesses typically focused their predictive modeling and even their optimization on the near term. Short-term considerations, such as Net Operating Income (NOI) or quarterly profitability, are often useful; however, it may be even more useful to enable decision-makers to project much further into the future, so that behaviors that do not completely manifest in a 6 or 12-month period may still be fully considered in the optimization process 200. Long-term forecasting may enable the manager to make decisions with a strategic focus. Integrated modeling and forecasting may enable the manager to balance short-term and long-term concerns to determine which decisions provide the most value to the business.

Figure 11a is a block diagram which illustrates an optimization process 200 with an objective function of lifetime customer value (LCV) according to one embodiment. Figure 11b is a dataflow diagram which illustrates an optimization process 200 with an objective function of lifetime customer value according to one embodiment. As discussed above, the method and system for optimization may be applied to select an optimal course of action for marketing products to the plurality of customers. In one embodiment, the objective function 230 may include

considerations of lifetime customer value. In other words, the objective function may be a function of LCV. In one embodiment, the LCV may be implemented through the use of constraints 235 in addition to the use of the objective function.

The particular function which expresses lifetime customer value may vary from optimization process to optimization process. For example, LCV may be calculated differently for a particular credit card offer optimization than for a particular pharmaceutical offer optimization. Generally, LCV may include a sum of discounted cash flows over time. In one embodiment, in other words, the basic form of LCV is as follows:

$$\mathrm{LCV} = \sum_{t} \frac{(\text{Net Income After Tax_t})}{(1 + \text{Discount Rate})^{t}} \tag{40}$$

wherein Net Income After Tax_t (NIAT) is a difference of cash inflows and cash outflows for a time period "t". Cash inflows for time period "t" may include, for example, sales income, fee income, and interest income. Cash ouflows for a time period "t" may include, for example, charge-off expense, cost of funds, and marketing cost. Net Income After Tax_t is therefore discounted and summed over the entire post-treatment period. In this way, the value that each customer in the portfolio will provide to the organization over the lifetime of the relationship may be found.

LCV may include a long-term view and a short-term view of the customer relationship(s). As used herein, a "long-term view" includes a period of one or more years. The combination of the long-term view and short-term view may be implemented by the use of both constraint(s) and objective to embody the LCV. For example, one or more constraints may be used to ensure that the business maintains a particular level of revenue over the short term, while the objective may be used to ensure that the business maximizes the value of the customer relationships over the long term. Likewise, the objective may include short-term considerations and the constraints may include long-term considerations. Various combinations of the objective, constraints, short-term considerations, and long-term considerations may be used to arrive at an appropriate LCV function for a particular optimization process 200.

In one embodiment, as shown in Figure 11b, the action variables 219 over the lifetime of the relationship may be received as input to a data calculator such as a cash flow calculator 232. The cash flow calculator 232 may include a set of financial formulas that are used to calculate the constraint function(s) and the objective function including LCV 234 for each input data set of the external variables 212 and decision variables 214. In one embodiment, the cash flow calculator 232 may calculate a series of per-customer LCV values c_{ij}^{s} for a given segment i, offer j, and scenario s, where the per-customer LCV is a function of the action variables a_{ij}^{s} for a given customer/segment i, offer j, and scenario s:

$$c_{ij}^{s} = LCV(a_{ij}^{s}) \tag{41}$$

The cash flow calculator 232 may measure all the cash inflows and outflows required by the objective function and constraints 234. The cash flow calculator 232 may calculate inflows and outflows from model predictions and forecasts of account behavior as well as from other data available to the decision-maker. In doing

so, the cash flow calculator 232 may convert available data into a series of detailed and customizable cash flows at particular time intervals (e.g., monthly, quarterly, bi-annually or annually). Cash inflows and outflows may also facilitate the calculation of "terminal value." Individual cash flows may be calculated at the most granular or detailed level with respect to the different individual components that comprise all flows of cash. Individual cash flows may be discounted and may ultimately form the LCV expression that is the objective function 230 of the optimization analysis.

Figure 12: Robust Optimization of Lifetime Customer Value with Cross-Record Constraints and/or Cross-Record Objective

Figure 12a is a block diagram which illustrates a robust optimization problem with cross-record constraints and an objective function of lifetime customer value (LCV) according to one embodiment. Figure 12b is a dataflow diagram which illustrates a robust optimization problem with cross-record constraints and an objective function of lifetime customer value (LCV) according to one embodiment. In other words, the approaches discussed with reference to Figures 3 through 11 may be practiced together according to one embodiment.

As shown in Figure 12a, an optimization process 200 may accept the following elements as input: customer information records 202, customer model(s) 204, uncertainty model(s) 260, one or more constraints 271, and an objective 231. The constraint(s) 271 and/or objective 231 may include LCV. The constraints 271 and/or objective 231 may be cross-record. The optimization process 200 may produce as output an optimized set of decision variables 210.

As shown in Figure 12b, the customer information 202 including decision variables 214 and external variables 212 may be input into the predictive model(s) 216 to generate the action variables 219 over the lifetime of the customer relationship. The predictive model(s) 216 may include the customer model(s) 204 and uncertainty model(s) 262. A data calculator such as a cash flow calculator 232 may include a set of financial formulas that are used to calculate the constraint function(s) and the objective function including LCV and cross-record constraints and/or cross-record objective 270. An optimizer 210 may then produce an optimized set of decision variables 210 which may be applied by the business to increase the value of customer relationships.

In the case of multiple customers with multiple products across multiple scenarios, a large number of models may be required. This population of models may be thought of as forming a cube of models, with the three axes representing customers/segments, products, and scenarios, respectively. For example, for a problem with 10 customers/segments, 10 products, and 20 scenarios, 2,000 models may be required. The form of the optimization for these problems is also generally the same as that for multiple customers/products. Therefore, in one embodiment, the multiple models may be included in the objective and constraints. Depending on the final form of the objective and constraints, the appropriate optimization technique may be selected, as discussed with reference to Figures 3 through 6.

Example 1: Linear Programming Formulation without Scenario Probabilities

An example of robust optimization (RO) of LCV with a cross-record constraint of Net Operating Income (NOI) is given as follows. In expressions (42) through (44), d are decision variables, d_{ij}^{s} are decision variables for segment i given offer j under scenario s, e are external variables, e_{ij}^{s} are external variables for segment i given offer

j under scenario s, a are action variables, a_{ij}^s are action variables for segment i given offer j under scenario s, o are objective parameters (such as penalty weights or objective coefficients), n_{ij} is the number of customers (i.e., current customers or prospective customers) in segment i who get offer j, c_{ij}^s is LCV per customer for segment i given offer j under scenario s, N_{ij}^s is NOI per customer in segment i given offer j under scenario s, N_t is an NOI requirement for time period t, and A_{ij}^s is the admissibility of offer j in segment i under scenario s ($A_{ij}^s = 1$ if admissible; $A_{ij}^s = 0$ otherwise). As discussed with reference to Figure 6, $c_{ij}^s = LCV(a_{ij}^s)$. This formulation has multiple scenarios such that one of the RO decision criteria discussed above could be applied to find the RO solution:

$$\max J = f(d,e,a,o) = \sum_i \sum_j c_{ij}^s n_{ij} A_{ij}^s \quad \text{for all } s \tag{42}$$

$$\text{such that } \sum_i \sum_j N_{ij}^s n_{ij} A_{ij}^s \geq N_t \quad \text{for all } s,\ t \tag{43}$$

This is a typical constraint which shows that in each scenario s and in each time period t, the total amount of near-term revenue (expressed here as Net Operating Income) must exceed some threshold value N_t. A similar constraint could keep losses below a given threshold. In fact, almost any resource may be constrained.

This formulation may be converted into a Linear Program (LP) by introducing the scalar variable y. In the formulation shown as expression (44), each scenario above becomes one constraint, and the variable y becomes the objective value of the LP. This LP gives the Maximin RO solution. The scenarios do not have probabilities.

$$\max \text{ over } n_{ij} \left\{ \begin{array}{l} y \text{ such that } \sum_i \sum_j c_{ij}^s n_{ij} A_{ij}^s \geq y \quad \text{for all s} \\ \qquad\qquad \sum_i \sum_j N_{ij}^s n_{ij} A_{ij} \geq N_t \quad \text{for all s, t} \end{array} \right\} \tag{44}$$

Example 2: Linear Programming Formulation with Scenario Probabilities

An example of robust optimization (RO) of LCV with "soft" cross-record constraints of minimum Net Operating Income (NOI) and maximum first-year loss, using slack variables, is given as follows. In expressions (45) through (49), p_s is the probability that scenario s occurs, E(.) is mathematical expectation (i.e., expected value), w_k is a weighting factor for constraint k, nd_k^s is negative deviation in scenario s from meeting constraint k, nd_k is the negative deviation across all scenarios, pd_k^s is positive deviation in scenario s from meeting constraint k, d are decision variables, d_{ij}^s are decision variables for segment i given offer j under scenario s, e are external variables, e_{ij}^s are external variables for segment i given offer j under scenario s, a are action variables, a_{ij}^s are action variables for segment i given offer j under scenario s, o are objective parameters (such as penalty weights or objective coefficients), n_{ij} is the number of customers (i.e., current customers or prospective customers) in segment i who get offer j, n_i is the number of accounts represented by record i (i.e., in that microsegment), c_{ij}^s is LCV per customer for segment i given offer j under scenario s, N_{ij}^s is NOI per customer in segment i given offer j under scenario s, N_k is NOI requirement number k, L_{ij}^s is first-year losses per customer in segment i given offer j under scenario s, L_k is

first-year loss requirement number k, A_{ij}^s is the admissibility of offer j in segment i under scenario s ($A_{ij}^s = 1$ if admissible; $A_{ij}^s = 0$ otherwise), and A_{ij} is the admissibility of offer j in segment i under all scenarios. As discussed with reference to Figure 6, $c_{ij}^s = LCV(a_{ij}^s)$. In this example, the LP formulation for maximizing the expected value of LCV is as follows:

$$\max J = f(d,e,a,o) = \sum_i \sum_j E(c_{ij}) n_{ij} A_{ij} - \sum_k E(nd_k) w_k \tag{45}$$

such that

$$\sum_i \sum_j N_{ij}^s n_{ij} A_{ij}^s = N_k + pd_k^s - nd_k^s \quad \text{for } k = 1 \text{ and all } s \tag{46}$$

$$\sum_i \sum_j L_{ij}^s n_{ij} A_{ij}^s = L_k - pd_k^s + nd_k^s \quad \text{for } k = 2 \text{ and all } s \tag{47}$$

As expressed in scenario constraint (46), in each scenario s, in each time period k, the expected NOI must meet the NOI goal N_k. As expressed in scenario constraint (47), in each scenario s, the expected first-year losses must meet (not exceed) the loss goal L_k.

$$\sum_j n_{ij} A_{ij} = n_i \tag{48}$$

$$\sum_i \sum_j n_{ij} [1 - A_{ij}] = 0 \tag{49}$$

As expressed in record/segment constraint (48), the total number of offers given within microsegment i must equal the number of accounts in the microsegment. As expressed in record/segment constraint (49), the number of inadmissible offers must be zero.

The objective function written with scenario probabilities may therefore be expressed as follows:

$$\max \sum_s p_s \left[\sum_i \sum_j (c_{ij}^s) n_{ij} A_{ij}^s - \sum_k nd_k^s w_k \right] \tag{50}$$

The objective function (50) may take several forms. It may use some function σ to reflect the tradeoffs among the scenarios. For example:

$$\sigma(\omega_1, \omega_2, \ldots, \omega_s) = \sum_s p_s \omega_s \quad (51)$$

$$\sigma(\omega_1, \omega_2, \ldots, \omega_s) = \sum_s p_s U(\omega_s) \quad (52)$$

$$\sigma(\omega_1, \omega_2, \ldots, \omega_s) = \min_s \omega_s \quad (53)$$

The expected value approach (51) is often called Stochastic Linear Programming. Problems with this form of the objective are the easiest to solve since they are linear, but are not always appropriate to a given problem. Such an objective implies risk neutrality on the part of the decision-maker. The utility function approach (52) allows the risk tolerance of the decision maker to be modeled explicitly, which is better for risk averse decision making, unless the LCV of the objective function have already been adjusted to account for risk. The utility function in the objective necessitates nonlinear optimization techniques. Finally, the Maximin formulation of the objective function (53) finds values for the decision variables that maximize LCV in the worst-case scenario. An alternate formulation is to maximize near-term profit while placing constraints on Expected LCV. The Maximin formulation may be appropriate when the decision-maker's risk aversion is strong. By selecting an appropriate formulation, a robust optimization problem may be solved in accordance with a customer's attitude regarding risk.

Figure 13: Optimization of Customer Relationship Management

Figure 13 is a flowchart which illustrates optimization of customer relationship management according to one embodiment. In step 302, a set of customer data corresponding to one or more customers may be collected and received. As discussed above, the customer data may include external variables and decision variables. In step 304, given the set of customer data collected in step 302, one or more decision variables may be optimized based on an objective and one or more constraints to produce optimized decision variables for a product offer. Step 304 may further include inputting the set of customer data from step 302 into one or more predictive models to generate one or more predictive customer behaviors (i.e., action variables). In one embodiment, the one or more predictive models may comprise a neural network. Step 304 is discussed in greater detail with reference to Figures 3 through 12.

As discussed above with reference to Figure 11, in one embodiment, the objective and/or constraints may comprise a lifetime customer value function. The lifetime customer value function may comprise a sum of expected cash flows over the lifetime of a relationship with the customer. The lifetime customer value function may represent a long-term view and a short-term view of the customer relationship. In one embodiment, a long-term view is a period of two or more years.

In one embodiment, optimizing the decision variables may further include optimizing the decision variables across a plurality of uncertainty models. As discussed in greater detail with reference to Figures 8 through 10, in one embodiment, the uncertainty models may comprise scenarios. The uncertainty models may be generated, for example, by varying a set of external variables to represent varying circumstances of possible futures.

In one embodiment, the constraints may include record-level constraints and/or cross-record constraints. Cross-record constraints include limitations on global resources. In one embodiment, the objective may be a cross-record objective. Cross-record constraints and objectives are discussed in greater detail with reference to Figure 7.

In step 306, the one or more terms of a product may be set to the optimized decision variables to produce an optimized product. The optimized product may comprise a set of optimized terms. For example, if product is a credit card, then the optimized terms may include an optimal interest rate and credit limit. Generally, setting the decision variables of the product to the optimized decision variables may result in increasing the value of the product vendor's relationships with customers of the product.

In step 308, the optimized product may be offered to one or more of the customers. In various embodiments, the optimized product may be offered to current and/or prospective customers in a variety of ways and under a variety of circumstances. For example, the optimized product may be offered via a direct mailing, targeted advertising, a re-pricing campaign, e-mail, and/or a promotional campaign for customer retention.

In step 310, a response from the customer to the offering may be accepted. In various embodiments, the response may be accepted automatically and without human intervention, such as through an automated on-line or web-based system, or the response may be accepted with human intervention.

In step 312, the optimized terms may be carried out during a lifetime of the optimized product. In other words, the offeror or product vendor may maintain the customer relationship in step 312 for a particular product according to the optimized terms produced in the optimization process of step 304. Carrying out the optimized terms may include conducting one or more transactions according to the optimized terms during the product lifetime. For example, a credit card issuer may honor the interest rate and credit limit of the optimized offer for one or more credit-card transactions for as long as the customer holds the credit card.

Software Architecture for Optimization of Customer Relationship Management

Figures 14-28 disclose an architecture for a software-based optimization and simulation system according to one embodiment. The software architecture may allow the generation of decisions based on an objective, constraints, and uncertainty models as discussed with reference to Figures 3 through 13. As discussed above, an objective is the target goal of a decision, a constraint is a qualifier for the objective, and an uncertainty model may include an expression of uncertainty regarding elements of the objective. In various embodiments, the decisions may be made in an off-line, proactive manner wherein many decisions are made all together (e.g., a batch method), or in an on-line, reactive manner wherein one decision is made at a time. The software architecture may solve the integration problems over the entire lifecycle of designing the decisions, making the decisions, and tracking the decisions.

For example, a decision may comprise determining an optimal annual percentage rate (APR) and credit line for each customer in a portfolio, wherein that APR and credit line yield the maximum lifetime customer value (LCV) with a minimum net operating income (for the following year) of $15 million. The objective in this decision is to maximize the lifetime customer value. This example optimization problem has one constraint: at least $15 million in net operating income for the following year. Furthermore, this example optimization problem has one risk factor or element of uncertainty: the cost of capital has the possibility of varying by ±10% over the next 5

years. Many other examples could be made with respect to other marketing decisions, manufacturing decisions, production decisions, and other suitable decisions.

To make such a decision most effectively, a holistic system may be used that may include designing experiments, modeling from the experiments and historical data, making the optimized decisions, tracking the results from the experiments/decisions, and embedding the experiments/decisions in on-line applications. One method of making the decision is in an off-line manner with a system that will make customer decisions in a batch manner. For example, this method might typically be used proactively to change the APR and credit line for all customers in a portfolio. Another method is in an on-line manner with a system that makes the decision for one customer at a time. This manner might typically be used to react to a particular event, such as the customer calling to request a change. The software architecture disclosed herein comprehends both off-line and on-line methods.

Figure 14: Off-line System Architecture

Figure 14 illustrates an off-line system architecture for optimization of customer relationship management according to one embodiment. In one embodiment, the architecture not only supports the ability to make off-line decisions, but also supports performing simulations prior to deploying the off-line and on-line decisions. There are four main sections to the architecture: design of experiments, tracking, model building and decision making. The sections may be roughly identified using the modules surrounding the four data stores 2360, 2400, 2530, 2680. The design of experiments section includes the Experiment Database 2360 and the four modules connected to it: Import 2100, Export 2150, Design Experiment 2300, and Experiment Engine 2340. The tracking section includes the Tracking Database 2400 and the Import Module 2100 connected to it. The model-building section includes the Model Database 2530 and the two modules connected to it: Import 2100 and Build Components 2500. The decision-making section includes the Decision Database 2680 and the six modules connected to it: Import 2100, Export 2150, Assemble/Setup Components 2600, Decision Views 2660, Decision Engine 2640, and Decision Reports 2670. Various Import 2100 and Export 2150 modules may be coupled to External Data 2200. As shown in Figure 14, the off-line system architecture may include Transformations 2120, Component Mathematical Models 2510, and Segmentation and Sampling Rules 2330.

The design of experiments section provides tools for creating and deploying experiments. The purpose of the experiments is to test various combinations of decision values (i.e., the values of decision variables) in preparation for modeling the results. For example, in a credit card re-pricing problem wherein the decision variables concern APR and credit line values for an account, an experiment may test combinations of APR and credit line values with current customers and prospective customers. The results may then be used in the tools provided by the tracking and model-building sections of the architecture.

The model-building section provides tools for creating and deploying mathematical models. As discussed above, these models may be built from (e.g., trained on) historical data as well as previous experiments and decisions. The mathematical models may encapsulate the data and results in a form that can be used to make future decisions. An example is a mathematical model calculating lifetime customer value of a credit card decision. Such models may be used in the tools provided by the decision-making section. Results from the models may also be tracked via the tracking section.

The decision-making section provides tools for creating and deploying optimized decision answers. For example, a pharmaceutical decision might include decision values concerning the number of drug samples and client visits. The optimal decision may include finding the right mix of samples and visits for the entire portfolio of clients, given budget constraints and risks related to uncertainty of travel costs. Results from the decisions may be used in the tracking section.

The tracking section provides tools for recording the results of the experiments deployed in the design of experiment section and the decisions deployed in the decision-making section. The results may then be used in the other sections to evaluate the performance of the experiments, models, and decisions. The Tracking Database 2400 may contain the experiment, model, and decision data that will be tracked. It may be organized by the projects that were created in the design of experiments, model-building, and decision-making sections. The various projects may be linked to associate them with one another. Thus, in one embodiment, one is able to know if a model-building project relates to a decision-making project and a design-of-experiment project. Furthermore, a given project might be associated with another project. For example, a given design-of-experiment project might be derived from another design-of-experiment project. Data may be put into the Tracking Database 2400 when a project is deployed. Deploying a project's experiment via the Export module 2150 may cause the experiment's data to be saved to the Tracking Database 2400. In one embodiment, each section has a sub-module that includes tools that use the Tracking Database 2400.

It is also conceived that these modules can be packaged in many forms. For example, they could be packaged as a desktop software product, as a client-server software product that supports multiple users, or as a web software product.

Figure 15: Import Module

Figure 15 illustrates the sub-modules of an Import module 2100 for optimization of customer relationship management according to one embodiment. The Import module 2100 may be used to load data into the various system databases. The Data Transformation Engine sub-module 2110 converts the data received from an external database or system to a usable form for the various tools using the system databases. Visualization Tool sub-modules 2140 may cooperate with the Data Transformation Engine sub-module 2110 to view the data and statistics of the data and build the transformations with those views. Example Visualization Tools 2140 are XY plots, row number plots, histogram plots, correlation plots, probability plots, PCA plots, and other suitable tools. The transformations can be built using the views and/or using an editor to explicitly express the transformations in terms of the data. In one embodiment, the Import module 2100 is extensible to add and remove Visualization Tool sub-modules 2140. The transformations can be saved in a reusable form 2120.

One potential reuse of the transformations is in an on-line application (as shown in Figure 27), wherein the transformations 2120 may be used to automatically transform the data from the on-line application. The application may use the Data Transformation Engine sub-module 2110 to transform the application data. The Import module 2100 may also contain Data Interface sub-modules 2130 for interfacing to the various system databases 2360, 2400, 2530, 2680 and to the various external databases and systems 2200. In one embodiment, the Import module 2100 is extensible to add and remove Data Interface sub-modules 2130.

Figure 16: Export Module

Figure 16 illustrates the sub-modules of an Export module 2150 for optimization of customer relationship management according to one embodiment. The Export module 2150 may be used to unload data from the various system databases. The Data Transformation Engine sub-module 2110 converts the data contained in the system databases to a usable form for the various external systems and databases. The same Data Transformation Engine sub-module 2110 may be used for the Export 2150 and Import 2100 modules. The Export modules 2150 contain Visualization Tool sub-modules that cooperate with the Data Transformation Engine sub-module 2110 to build the transformations. In one embodiment, the Export module 2150 is extensible to add and remove Visualization Tool sub-modules. The Visualization Tool sub-modules 2140 used in the Export module 2150 may or may not be the same as those used in the Import module 2100. In one embodiment, however, all Visualization Tool sub-modules 2140 support the same interface to be used in either. The transformations built can be saved in a re-usable form 2120.

One potential reuse of the transformations is in an on-line application (as shown in Figure 27), wherein the transformations 2120 may be used to automatically transform the data from the on-line application. The application may use the Data Transformation Engine sub-module 2110 to transform the data in the Experiment Database 2360. The Export module 2150 also contains Data Interface sub-modules 2130 for interfacing to the various system databases 2360, 2400, 2530, 2680 and the various external databases and systems 2200. In one embodiment, the Export module 2150 is extensible to add and remove Data Interface sub-modules 2130. The Import 2100 and Export 2150 modules may share some of the same Data Interface sub-modules 2130.

Figure 17: Design Experiment Module

Figure 17 illustrates the sub-modules of the Design Experiment module 2300 for optimization of customer relationship management according to one embodiment. The Design Experiment module 2300 may be used to create, delete, edit, view, and track the experiments. The Design Experiment module 2300 includes visualization tools that work with the Experiment Database 2360 and Experiment Engine 2340 to create, delete, edit, and view the experiments. The Create Tool sub-module 2310 creates experiments for a project. The Delete Tool sub-module 2315 deletes experiments of a project. The Edit Tool sub-modules 2305 manage the segmentation and sampling rules 2330 used by the Experiment Engine 2340. The View Tool sub-modules 2320 include various visualizations of the experiments, including reports and charts showing the data after the segmentation and sampling rules are applied. The Track Tool sub-modules 2325 include reports and charts comparing the results of executing the experiment to what was designed. In one embodiment, the Design Experiment module 2300 is extensible to add and remove Edit Tool 2305, View Tool 2320, and Track Tool 2325 sub-modules. The Design Experiment modules also includes functionality to manage the projects that contain the experiments.

Figure 18: Experiment Engine Module

Figure 18 illustrates the sub-modules of the Experiment Engine module 2340 for optimization of customer relationship management according to one embodiment. The Experiment engine module 2340 may be used to apply the segmentation and sampling rules 2330 against the transformed data in the Experiment Database 2360. The Segment Engine sub-module 2350 may encapsulate a segmentation algorithm and perform that algorithm

according to the segmentation rules created in the Design Experiment module 2300. The segmentation rules may be specific to the segmentation algorithm. Segmentation algorithms may comprise simple cut rules to complicated clustering algorithms. The Sample Engine sub-module 2350 may encapsulate a sampling algorithm and perform that algorithm according to the sampling rules created in the Design Experiment module 2300. The sampling rules may be specific to the sampling algorithm. Sampling algorithms may include a variety of algorithms, from simple and random to complicated and score-based algorithms. In one embodiment, the Experiment Engine module 2340 is extensible to add and remove Segment Engine 2350 and Sample Engine 2345 sub-modules.

Figure 19: Build Components Module

Figure 19 illustrates the sub-modules of the Build Components module 2500 for optimization of customer relationship management according to one embodiment. The Build Components module 2500 may be used to build component mathematical models 2510 for use in the decision-making section. The component models may be of various types, such as neural network, logistic regression, linear regression, genetic algorithm, table lookup, and other suitable models. Projects may be created in the Build Components module 2500 to encapsulate the work for one or more component models. Each type of component model may have its own set of Edit Tools 2505, Analysis Tools 2515, and Track Tools 2520. The Edit Tools provide mechanisms for creating and editing component instances of the corresponding type of model. For example, an Edit Tool 2505 for a neural network component might identify data elements in the Model Database 2530 as inputs and outputs for the neural network and then train the neural network on the corresponding data. The Analysis Tools 2515 may provide mechanisms for analyzing and viewing the component models. For example, an Analysis Tool 2515 for a neural network might identify the sensitivities of the inputs to outputs. Another Analysis Tool 2515 for a neural network might show the internal layers with their corresponding weights. The Track Tools 2520 may provide mechanisms for comparing results from deploying the component models. For example, a Track Tool 2520 for a neural network would show predicted versus actual values for the outputs. In one embodiment, the Build Components module 2500 is extensible to add and remove Edit Tool 2505, Analysis Tool 2515, and Track Tool 2520 sub-modules.

Figure 20: Assemble/Setup Components Module

Figure 20 illustrates the sub-modules of the Assemble/Setup Components module 2600 for optimization of customer relationship management according to one embodiment. The Assemble/Setup Components module 2600 may be used to perform setup steps required prior to performing the optimization and simulation. Projects are created in the Assemble/Setup Components module 2600 to encapsulate the configuration work for one or more configuration variations. A configuration variation is encapsulated in an abstraction called a question. Questions may reference objectives managed by the Assemble Objective 2610 sub-module, risk variables managed by the Edit Risk 2615 sub-module, and segments managed by the Edit Segments 2620 sub-module. The Assemble Objective sub-module 2610 may be used to assemble component mathematical models 2510 into objectives for a project. Assembling may include binding component outputs to component inputs. Assembling may also include binding component inputs and outputs to data elements in the Decision Database 2690. The project can contain many objectives. Questions reference the project's objectives as desired by the user.

The Edit Risk sub-module 2615 may be used to create and edit the set of risk variables for a project. Risk variables are selected from the set of component inputs, component outputs, and Decision Database 2680 data elements. Risk variables may have properties of high and low value. The high and low values are expressed in relation to the actual value, such as a percentage difference, absolute value, etc. Questions reference the project's risk variables as desired by the user. The Edit Segments sub-module 2620 may be used to define the segmentation to be used for reporting and question solving. The segmentation may be defined according to dimensions, attributes, elements, and facts, which are concepts defined in Multi-Dimensional Databases. In addition, the Data Transformation Engine sub-module 2110 may be used in the Edit Segments sub-module 2620 to define transformations 2120 that build facts for the segmentation. These facts form the decision-level records to be used when solving the questions.

Figure 21: Manage Questions Sub-module

Figure 21 illustrates a hierarchy of the Manage Questions sub-module 2605 for optimization of customer relationship management according to one embodiment. In one embodiment, questions may be of two base types: optimization and prediction. An optimization question solves for a set of input variables in an objective. A prediction question calculates an objective. There may also be derivations of these two base types. For example, marginal optimization includes an additional objective called the baseline to compare against. In one embodiment, all questions have basic functionality provided by the Manage Questions sub-module 2605. This functionality includes creating, copying, and deleting questions. The functionality also includes selecting from the project an objective, the risk variables, and the segmentation element(s) to use for solving the question. The Manage Prediction sub-module 2606 adds the functionality of selecting additional objectives. The Manage Optimization 2607 and Manage Marginal Optimization 2608 sub-modules are discussed below.

Figure 22: Manage Optimization Sub-module

Figure 22 illustrates the sub-modules of the Manage Optimization sub-module 2607 for optimization of customer relationship management according to one embodiment. The Manage Optimization sub-module 2607 may add functionality for managing decision variables, constraints, and reason codes to the Manage Questions sub-module 2605. The Edit Decision Variable sub-module 2625 may be used to create and edit the set of decision variables for a project. Decision variables have properties related to their values, and the properties are specific to the type of variable. In one embodiment, two types of decision variables may be used: discrete and continuous. Discrete variables have properties describing the possible values. These values can be expressed, for example, as a minimum value, maximum value, and increment; or they can be expressed as a set of discrete values. Continuous variables have properties of minimum and maximum value. The Manage Optimization sub-module 2607 manages the subset of project's decision variables to solve for in a question. The Manage Optimization sub-module 2607 also manages the allowable decision variable value combinations for the question.

The Edit Constraints sub-module 2630 may be used to create and edit the optimization constraints for a project. Constraints are expressed using terms that include data constants, data elements in the Decision Database 2680, expressions containing data elements, and/or data constants. The constraint expression may include three parts: a left-hand term, a relational operator, and a right-hand term. Relational operators are the typical numeric

relational operators such as equal, less than, greater than, etc. Constraints can be global, regional, or local. As discussed above, global and regional constraints are constraints where the terms used rolled-up facts from the segmentation. For example, a global constraint could be total net operating income over the entire portfolio greater than $15 million; and a regional constraint could be total mail costs in the southwest less than $1 million. Local constraints are constraints where the terms use facts from the decision-level records. For example, a local constraint could be number of mailings greater than 0. The Manage Questions sub-module 2605 manages the subset of a project's constraints to use in solving the question.

The Edit Reasons sub-module 2635 may be used to create and edit the rules for generating reason codes associated with the solved decision variables. The rules are expressions that if true, produce a reason code. The expressions are built using terms that may include data constants, data elements in the Decision Database 2680, and/or results of calculations on those data elements. The terms may be put together as relational and logical expressions. For example, such expressions may include "BehaviorScore < 600" and "CurrentCL > NewCL".

Figure 23: Manage Marginal Optimization Sub-module

Figure 23 illustrates the sub-modules of the Manage Marginal Optimization sub-module 2608 for optimization of customer relationship management according to one embodiment. The Manage Marginal Optimization sub-module 2608 may add the functionality of managing a baseline objective to the Manage Optimization sub-module 2608. The Manage Baseline sub-module 2636 identifies the objective from the project's list of objectives and initiates the calculation of the baseline objective for the decision records. A baseline objective is of the same structure as the optimization objective, except the behavioral components of the objective represent the normal business behavior. For example, if the optimization objective is lifetime customer value, the baseline objective is also lifetime customer value. And, the difference between the two is the set of components assembled into each.

Figure 24: Decision Engine Module

Figure 24 illustrates the sub-modules of the Decision Engine module 2640 for optimization of customer relationship management according to one embodiment. The Decision Engine module 2640 may be used to perform the calculations for a question. The Calculation Engine sub-module 2645 may be used to calculate the objective, constraints, and any miscellaneous equations for a decision level record. The objective, constraints, and miscellaneous equations may be created in the Assemble/Setup Components module 2600 by assembling components. A decision-level record is either formed via the segmentation transformations in the Edit Segments sub-module 2635 or assumes the account-level records brought into the Decision Database 2680. An account-level record represents the data elements for a customer or some entity that is the focus of the decision.

The Decision Engine module 2640 may be configured with multiple Calculation Engine sub-modules 2645 depending on the configuration of the Question Engine sub-module 2650. The Question Engine sub-module 2650 performs the algorithm specific to a question. If the question is configured to perform an optimization, the Question Engine 2650 may include that optimization algorithm. If the question is configured to perform a prediction, the Question Engine 2650 may include that prediction algorithm. For example, if the question is configured to perform an optimization of discrete decision variables with global constraints and risk variables, then a Question Engine

2650 with that ability is used. In one embodiment, the Decision Engine module 2640 is extensible to add and remove Question Engine sub-modules 2650. It is also conceived the Decision Engine module 2640 could be built to run calculations in parallel or in a distributed manner. For example, the Question Engine sub-module 2650 could invoke multiple Calculation Engine sub-modules 2645 in parallel or across a network (in a distributed manner).

Figure 25: Decision Views Module

Figure 25 illustrates the sub-modules of the Decision Views module 2660 for optimization of customer relationship management according to one embodiment. The Decision Views module 2660 may be used to perform minor setup changes to a question, to start the calculations, and to view the results of the calculations. Minor setup changes include selecting the segment over which the calculations are run and selecting the risk variables. Access may also be provided to the setup functionality in the Assemble/Setup Components module 2600, which allows more major changes. Once the setup changes have been made, a mechanism is provided to start the calculations, which will be run by the Decision Engine module 2640. After the calculations have been run, various tables and charts are used to evaluate the decisions. The tables and charts are encapsulated via a View sub-module 2665. In one embodiment, the tables and charts have direct access to the results of the Decision Engine 2640, including details that might not be stored persistently in the Decision Database 2680. The View sub-modules 2665 may allow one to compare the scenario simulations generated from the risk variables and look at the robustness of the decisions across those scenarios. Access may also be provided to the reports 2675 in the Decision Reports modules 2670. In one embodiment, the Decision Views module 2660 is extensible to add and remove View sub-modules 2665.

Figure 26: Decision Reports Module

Figure 26 illustrates the sub-modules of the Decision Reports module 2670 for optimization of customer relationship management according to one embodiment. The Decision Reports module 2670 may be used to examine the results of a project's question from the Decision Database 2680. The results may be shown as reports via Report sub-modules 2675. These reports may take many forms, including web pages. In one embodiment, all reports may show the results within the segmentation. The results shown may include the decision mix, the constraint adherence, the objective values, intermediate calculations, and question configuration information. There may also be provided comparison reports that compare any of the results between projects and between questions. There may also be provided tracking reports showing the actual results versus the predicted results. The tracking reports work off the Tracking Database 2400. In one embodiment, the Decision Reports module 2670 is extensible to add and remove Report sub-modules 2675.

Figures 27 and 28: On-Line Architecture

In one embodiment, the system architecture for on-line, reactive decisions may be a modification of the off-line system architecture. On-line applications are usually built to serve other purposes than making decisions. Therefore, the process of making a decision needs to be embedded in those on-line applications. Figure 27 shows the system architecture for embedding the experiment process according to one embodiment. Figure 28 shows the system architecture for embedding the decision process according to one embodiment. To embed the experiment

and decision processes, the application programming interfaces (APIs) for the Data Transformation Engine sub-module 2110, the Experiment Engine module 2340, and the Decision Engine module 2640 may be provided to the programmer of the on-line application.

Figure 27 illustrates an on-line experiment architecture for optimization of customer relationship management according to one embodiment. To embed the experiment process, the on-line application may be modified to use the APIs for the Data Transformation Engine sub-module 2110 and the Experiment Engine module 2340. The transformations 2120 and rules 2330 are built with the modules of the off-line system architecture (see Figure 14). Once built and tested via simulations, they may be deployed to the on-line application. The Experiment Database 2360 may be implemented as an in-memory database. In one embodiment, it is only required to hold the data elements for one customer or entity.

Figure 28 illustrates an on-line decision architecture for optimization of customer relationship management according to one embodiment. To embed the decision process, the on-line application may be modified to use the APIs for the Data Transformation Engine sub-module 2110 and the Decision Engine module 2640. The transformations 2120 and component models 2510 are built with the modules of the off-line system architecture (see Figure 14). Once built and tested via simulations, they may be deployed to the on-line application. If the decision does not include global or regional constraints, the Decision Database 2680 may be implemented as an in-memory database, since it is only required to hold the data elements for one customer or entity. However, with global or regional constraints, the Decision Database 2680 holds the data elements for all customers or entities to which the constraints apply.

Various embodiments may further include receiving or storing instructions and/or data implemented in accordance with the foregoing description upon a carrier medium. Suitable carrier media may include storage media or memory media such as magnetic or optical media, e.g., disk or CD-ROM, as well as signals such as electrical, electromagnetic, or digital signals, conveyed via a communication medium such as networks 102 and/or 104 and/or a wireless link.

Although the system and method of the present invention have been described in connection with several embodiments, the invention is not intended to be limited to the specific forms set forth herein, but on the contrary, it is intended to cover such alternatives, modifications, and equivalents as can be reasonably included within the spirit and scope of the invention as defined by the appended claims.

What is claimed is:

1. A method for selecting an optimal course of action for marketing products to a plurality of customers, the method comprising:

inputting customer data records into one or more predictive models to generate one or more action variables, wherein the action variables comprise predictive customer behaviors corresponding to the customer data records;

inputting an objective function and one or more constraints into an optimizer, wherein the constraints comprise limitations on one or more resources, and wherein the objective function and the constraints comprise a function of the action variables across a plurality of uncertainty models;

the optimizer solving the objective function subject to the constraints to select an optimal course of action across the plurality of uncertainty models for marketing products to a plurality of customers.

2. The method of any of the preceding claims, further comprising:

a user selecting the uncertainty models according to managerial judgment.

3. The method of claim 2,

wherein the user selecting the uncertainty models according to managerial judgment further comprises the user selecting a prior distribution.

4. The method of any of the preceding claims, further comprising:

generating the plurality of uncertainty models by sampling a probability distribution to represent varying circumstances of possible futures.

5. The method of any of the preceding claims,

wherein the uncertainty models comprise scenarios.

6. The method of claim 5, further comprising:

assigning a weight to each of the scenarios to denote its likelihood of occurrence.

7. The method of any of the preceding claims,

wherein the uncertainty models are implemented by a neural network.

8. The method of any of the preceding claims,

wherein the uncertainty models comprise one or more averaged models and a residual model.

9. The method of claim 8, further comprising:
training a neural network to represent the averaged models.

10. The method of any of the preceding claims,
wherein solving the objective function subject to the constraints to select the optimal course of action across the plurality of uncertainty models further comprises solving the objective function to avoid a worst case scenario.

11. The method of any of the preceding claims,
wherein solving the objective function subject to the constraints to select the optimal course of action across the plurality of uncertainty models further comprises solving the objective function according to a desired level of risk.

12. The method of any of the preceding claims, further comprising:
marketing the products to the plurality of customers in accordance with the optimal course of action.

13. The method of any of the preceding claims, further comprising:
applying the optimal course of action to one or more customer relationships to increase the value of the customer relationships.

14. The method of any of the preceding claims,
wherein each customer data record corresponds to an individual customer.

15. The method of any of the preceding claims,
wherein each customer data record corresponds to a segment, wherein the segment comprises a plurality of customers.

16. The method of any of the preceding claims,
wherein each customer data record corresponds to a sample of a plurality of customers.

17. The method of any of the preceding claims,
wherein the predictive models comprise a neural network.

18. The method of any of the preceding claims, further comprising:
training the neural network as a function of historical data records.

19. The method of any of the preceding claims,
wherein the optimizer solving the objective function further comprises solving the objective function via linear programming.

20. The method of any of the preceding claims,
wherein the optimizer solving the objective function further comprises solving the objective function via non-linear programming.

21. The method of any of the preceding claims,
wherein the objective function is a cross-record objective function.

22. The method of any of the preceding claims, further comprising:
inputting one or more cross-record constraints into the optimizer. wherein the cross-record constraints comprise limitations on one or more global resources;
and wherein the optimizer solving the objective function further comprises the optimizer solving the objective function subject to the cross-record constraints.

23. The method of any of the preceding claims,
wherein the objective function further comprises a function of lifetime customer value.

24. The method of claim 23,
wherein lifetime customer value comprises a sum of discounted cash flows over the lifetime of a customer relationship.

25. A system comprising:
one or more predictive models operable to generate one or more action variables from a plurality of customer data records, wherein the action variables comprise predictive customer behaviors corresponding to the customer data records; and
an optimizer coupled to the predictive models, wherein the optimizer is operable to receive an objective function and one or more constraints, wherein the constraints comprise limitations on one or more resources, wherein the objective function and the constraints comprise a function of the action variables across a plurality of uncertainty models, and wherein the optimizer is operable to solve the objective function subject to the constraints to select an optimal course of action across the plurality of uncertainty models for marketing products to a plurality of customers;
wherein the system is operable to perform the method of any of claims 1 through 24.

26. A carrier medium comprising program instructions which are computer-executable to implement the method of any of claims 1 through 24.

27. A method comprising:

inputting data records into one or more predictive models to generate one or more action variables;

inputting an objective function and one or more constraints into an optimizer, wherein the constraints comprise limitations on one or more resources, and wherein the objective function and the constraints comprise a function of the action variables across a plurality of uncertainty models;

the optimizer solving the objective function subject to the constraints to select an optimal course of action across the plurality of uncertainty models.

28. A method for determining an optimal course of action for marketing one or more products to a plurality of customers, the method comprising:

inputting customer data records into one or more predictive models to generate one or more action variables, wherein the action variables comprise predictive customer behaviors corresponding to the customer data records;

inputting an objective function and one or more constraints into the optimizer, wherein the objective function comprises a function of the action variables, wherein the constraints comprise limitations on one or more resources, and wherein the objective function further comprises a function of lifetime customer value;

the optimizer solving the objective function subject to the constraints to select an optimal course of action for marketing one or more products to a plurality of customers.

29. A method for selecting an optimal course of action for marketing one or more products to a plurality of customers, the method comprising:

inputting customer data records into one or more predictive models to generate one or more action variables, wherein the action variables comprise predictive customer behaviors corresponding to the customer data records;

inputting an objective function and one or more constraints into an optimizer, wherein the objective function comprises a function of the action variables, and wherein the constraints comprise limitations on one or more resources;

the optimizer solving the objective function subject to limitations on one or more global resources to select an optimal course of action for marketing one or more products to a plurality of customers.

30. A method for managing customer relationships, the method comprising:

collecting a set of customer data corresponding to one or more customers;

optimizing one or more decision variables based on an objective and one or more constraints for the set of customer data to produce optimized decision variables for a product offer;

setting one or more terms of a product to the optimized decision variables to produce an optimized product, wherein the optimized product comprises a set of optimized terms;

offering the optimized product to one or more of the customers;

accepting a response from the customer to the offering;

carrying out the optimized terms during a lifetime of the optimized product.

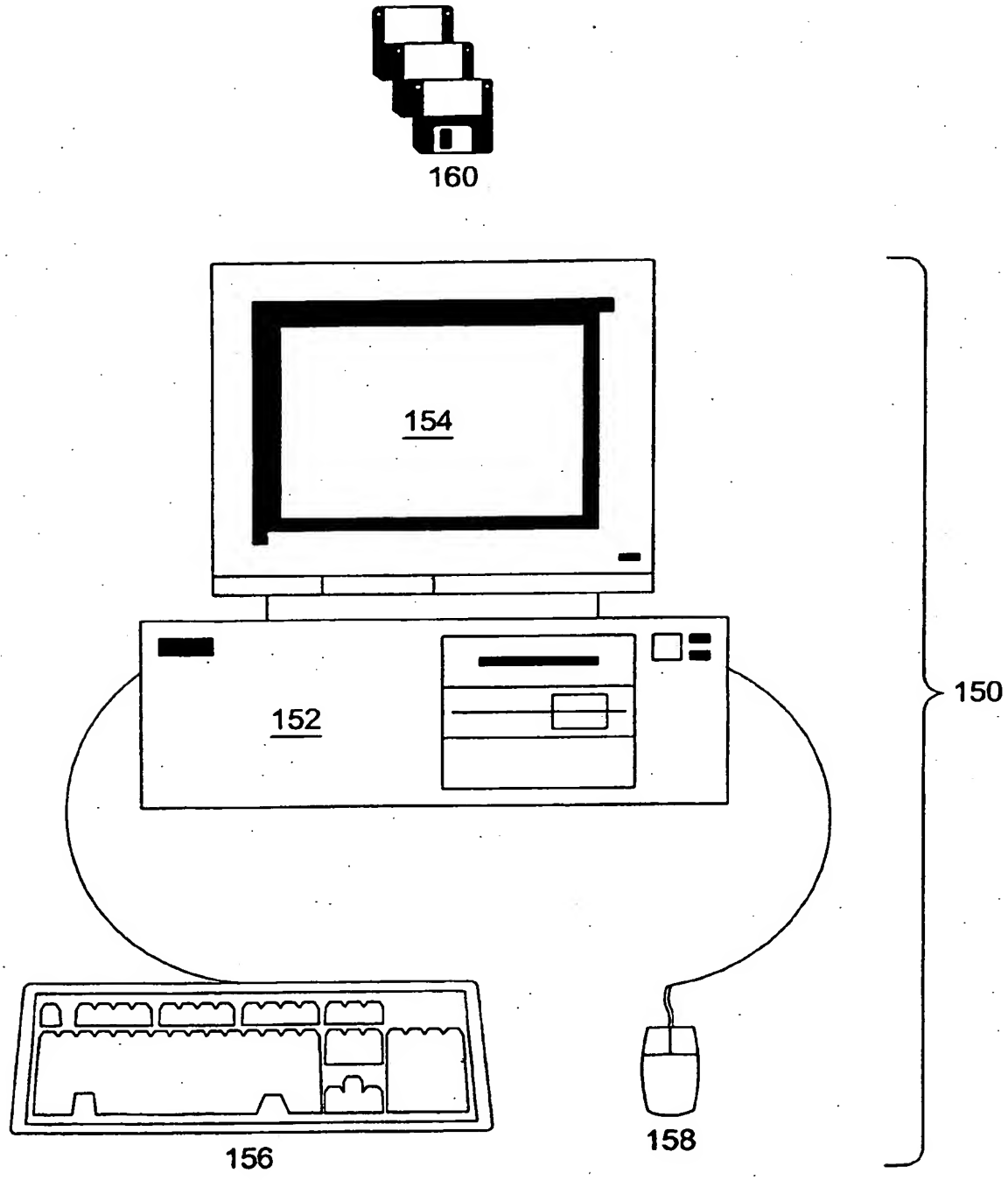

FIG. 1

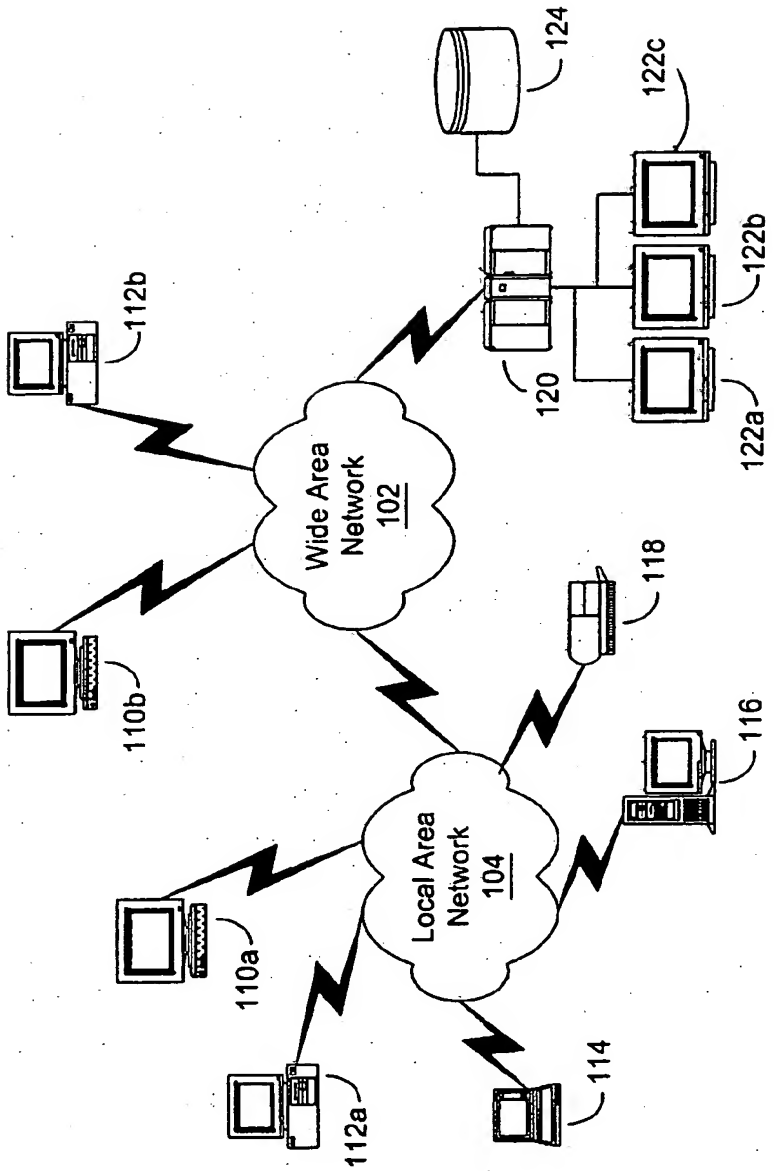
124
122c
122b
122a
120
112b
110b
Wide Area Network 102
118
116
Local Area Network 104
110a
112a
114

FIG. 2

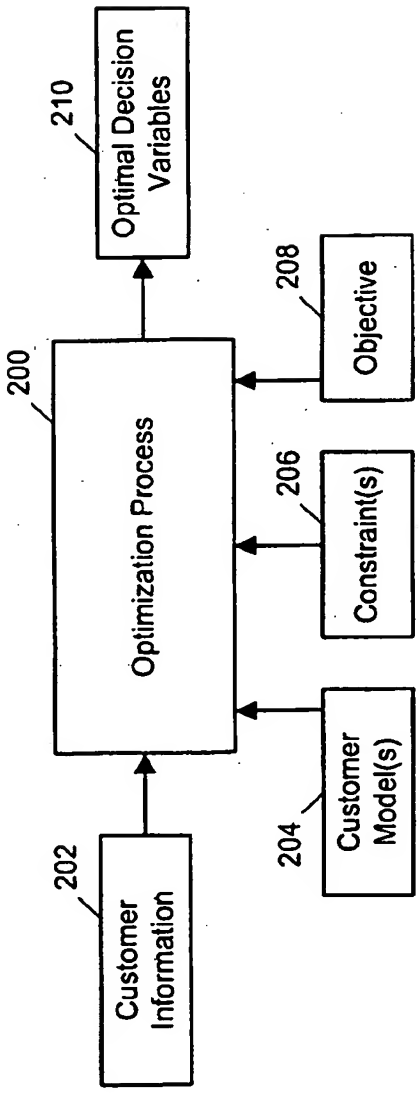
Customer Information
202
Customer Model(s)
204
Constraint(s)
206
Objective
208
Optimization Process
200
Optimal Decision Variables
210

FIG. 3a

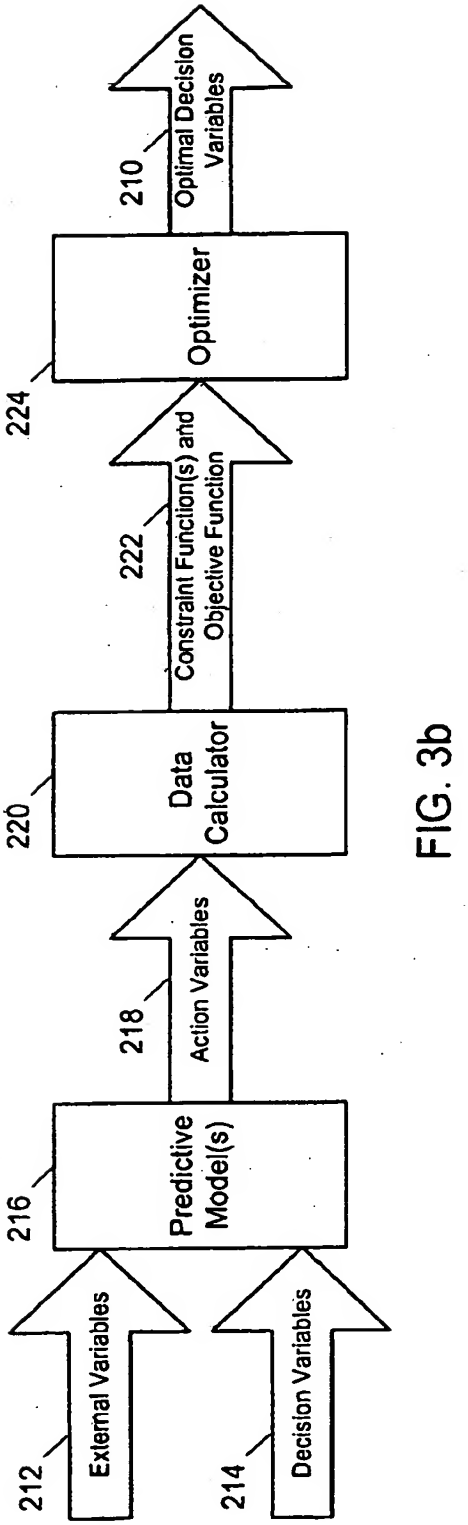
External Variables
212
Decision Variables
214
Predictive Model(s)
216
Action Variables
218
Data Calculator
220
Constraint Function(s) and Objective Function
222
Optimizer
224
Optimal Decision Variables
210

FIG. 3b

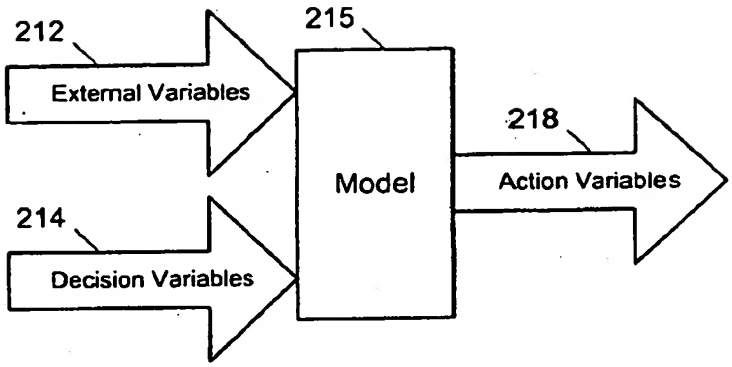
212
External Variables
215
Model
218
Action Variables
214
Decision Variables

FIG. 4

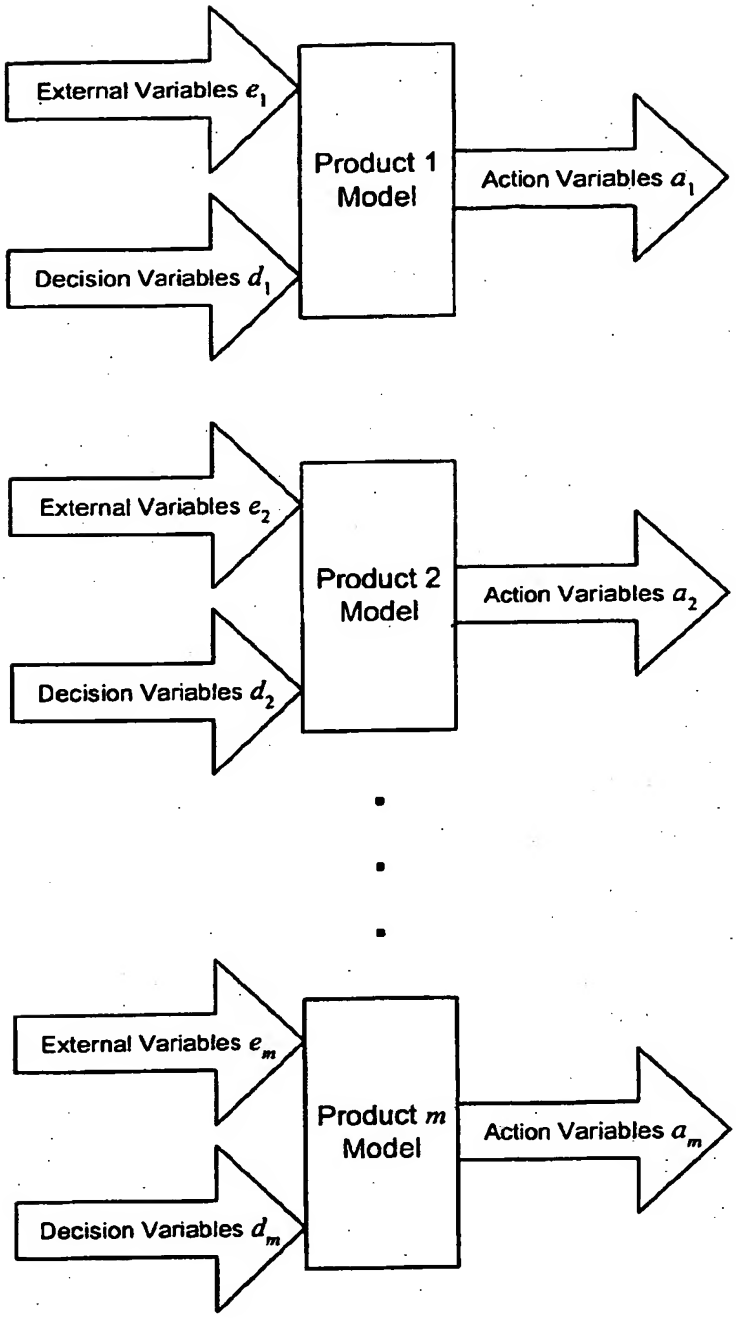
External Variables e_1
Decision Variables d_1
Product 1 Model
Action Variables a_1
External Variables e_2
Decision Variables d_2
Product 2 Model
Action Variables a_2
External Variables e_m
Decision Variables d_m
Product m Model
Action Variables a_m

FIG. 5

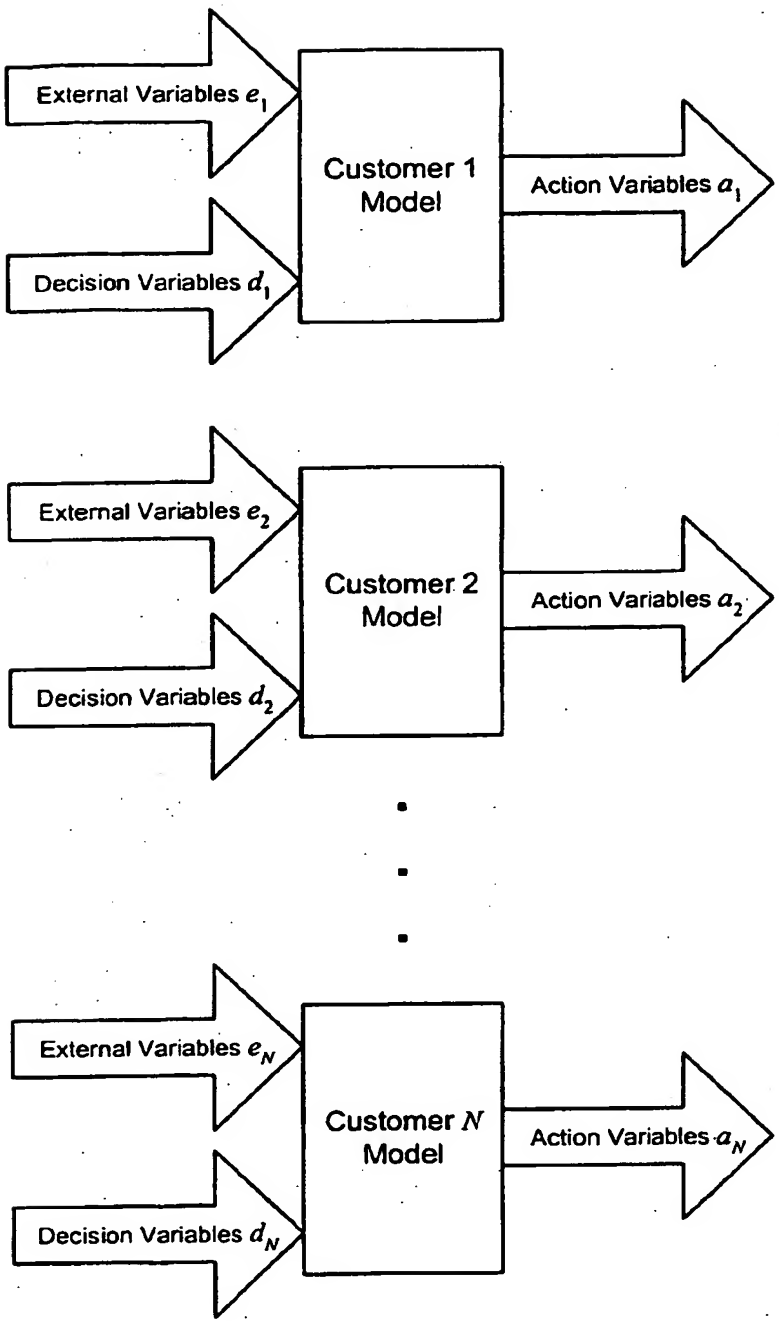
External Variables e_1
Decision Variables d_1
Customer 1 Model
Action Variables a_1
External Variables e_2
Decision Variables d_2
Customer 2 Model
Action Variables a_2
External Variables e_N
Decision Variables d_N
Customer N Model
Action Variables a_N

FIG. 6

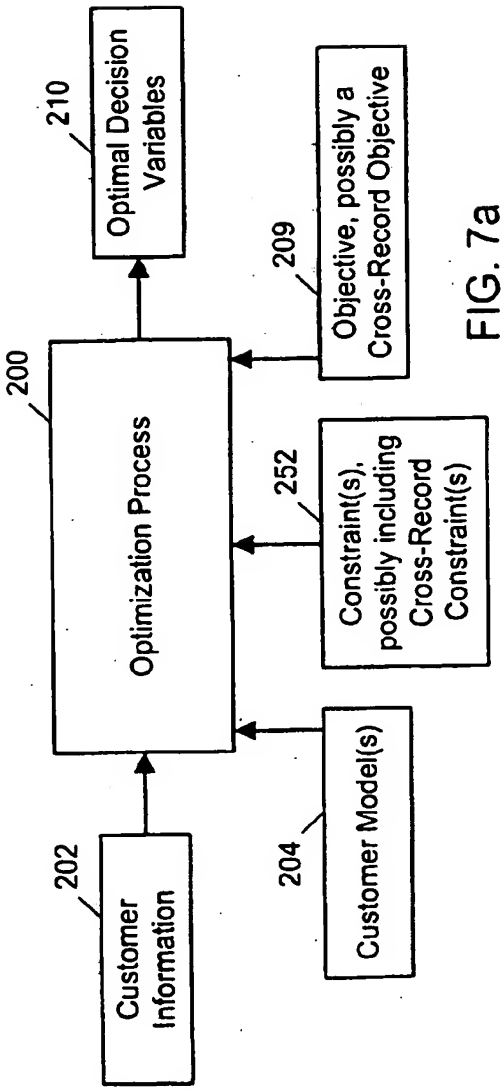
Optimal Decision Variables
210
Objective, possibly a Cross-Record Objective
209
Optimization Process
200
Constraint(s), possibly including Cross-Record Constraint(s)
252
Customer Information
202
Customer Model(s)
204

FIG. 7a

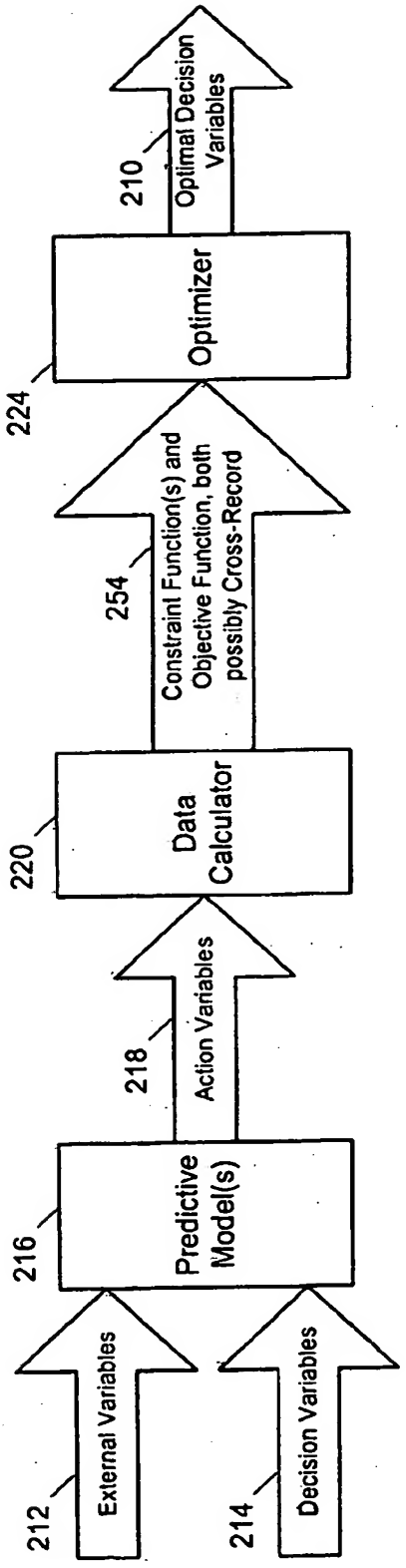
Optimal Decision Variables
210
Optimizer
224
Constrainl Function(s) and Objective Function, both possibly Cross-Record
254
Data Calculator
220
Action Variables
218
Predictive Model(s)
216
External Variables
212
Decision Variables
214

FIG. 7b

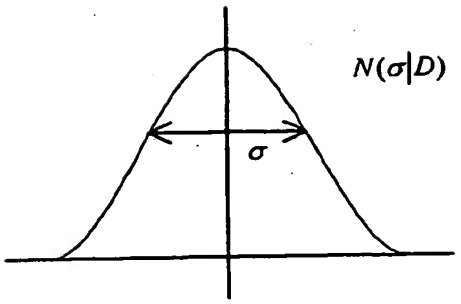

FIG. 8a

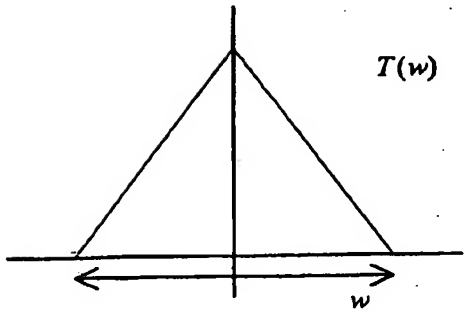

FIG. 8b

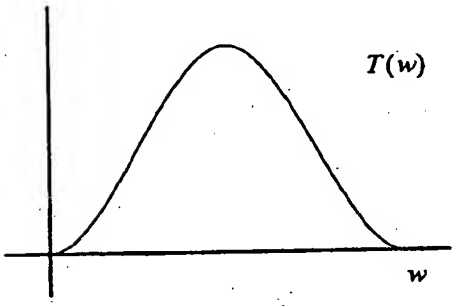

FIG. 8c

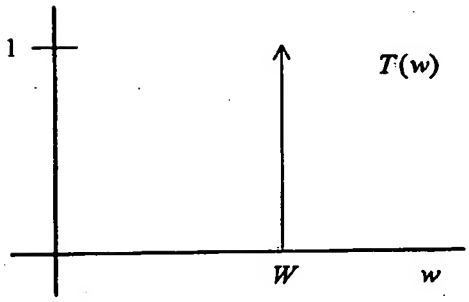

FIG. 8e

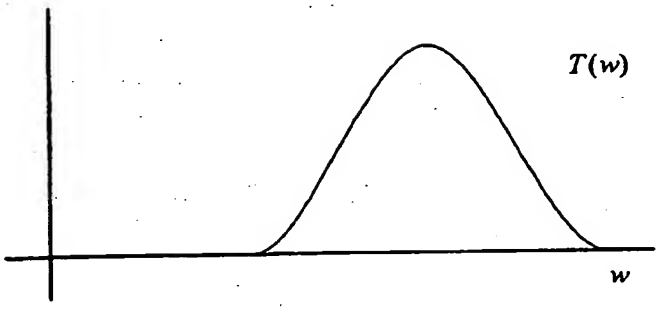

FIG. 8d

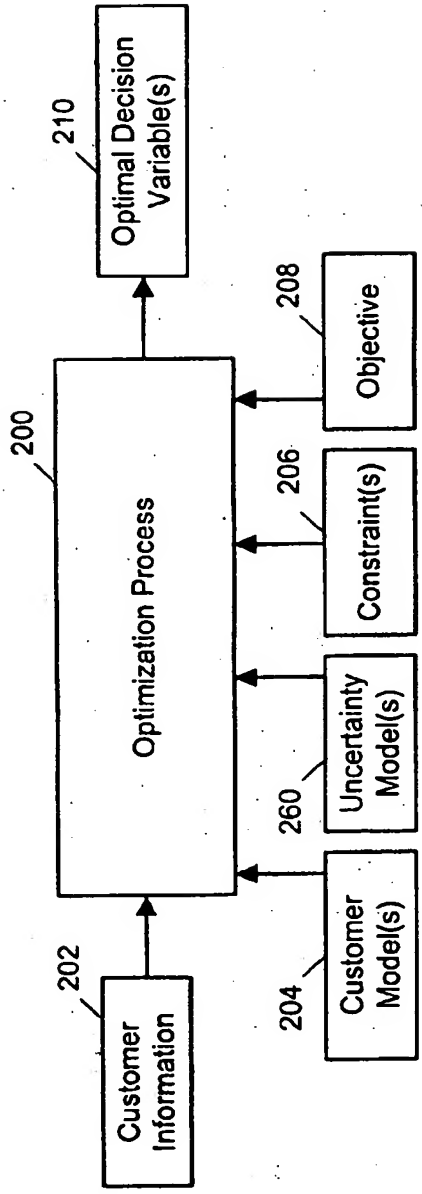
Customer Information
202
Customer Model(s)
204
Uncertainty Model(s)
260
Constraint(s)
206
Objective
208
Optimization Process
200
Optimal Decision Variable(s)
210

FIG. 9a

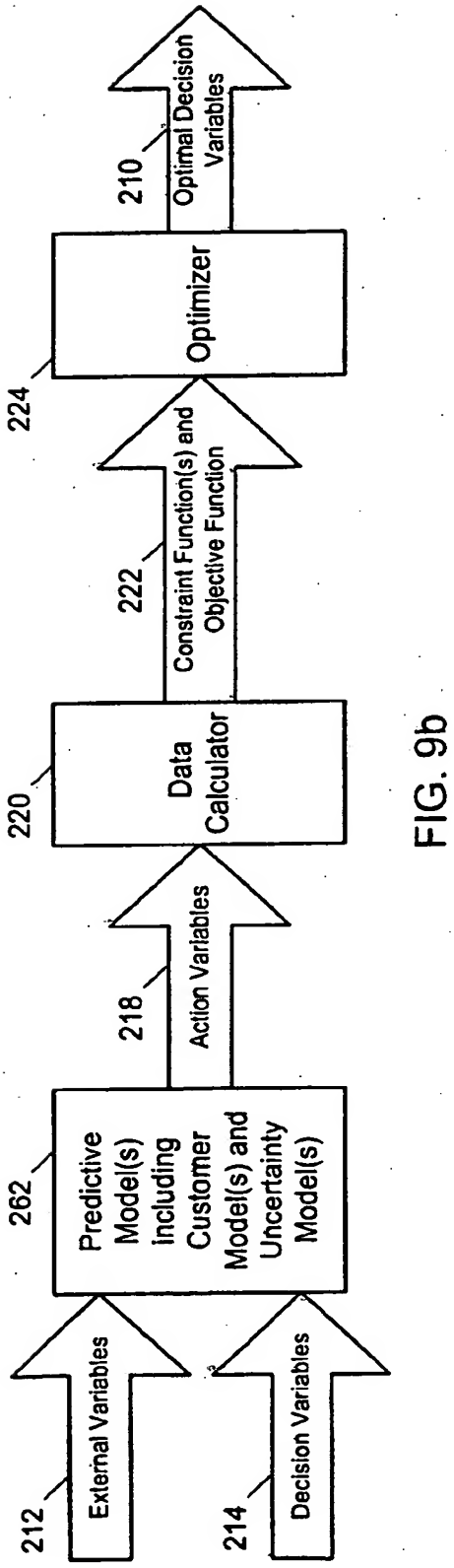
External Variables
212
Decision Variables
214
Predictive Model(s) including Customer Model(s) and Uncertainty Model(s)
262
Action Variables
218
Data Calculator
220
Constraint Function(s) and Objective Function
222
Optimizer
224
Optimal Decision Variables
210

FIG. 9b

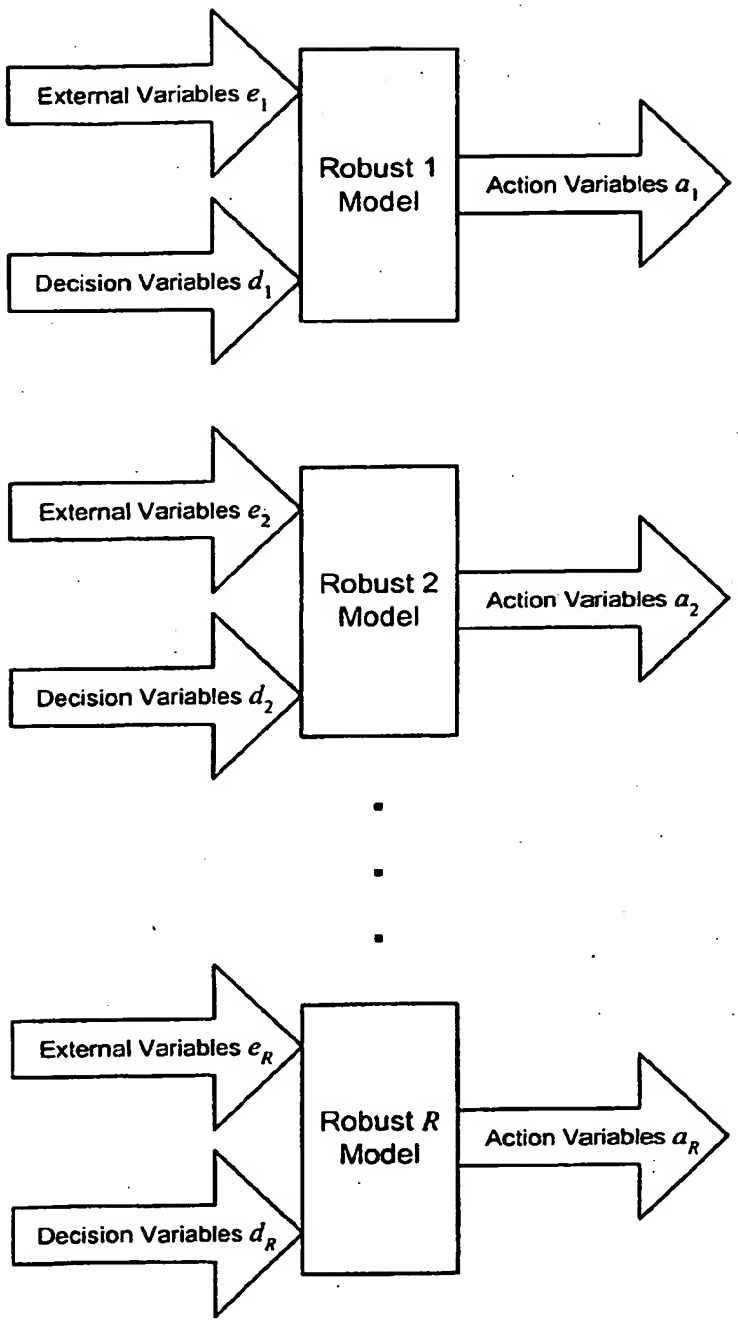
External Variables e_1
Decision Variables d_1
Robust 1 Model
Action Variables a_1
External Variables e_2
Decision Variables d_2
Robust 2 Model
Action Variables a_2
External Variables e_R
Decision Variables d_R
Robust R Model
Action Variables a_R

FIG. 10

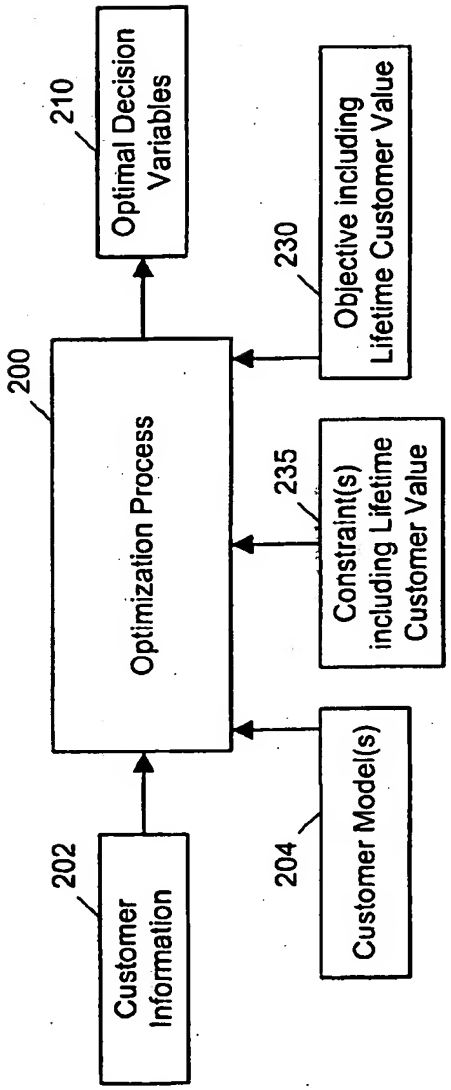

FIG. 11a

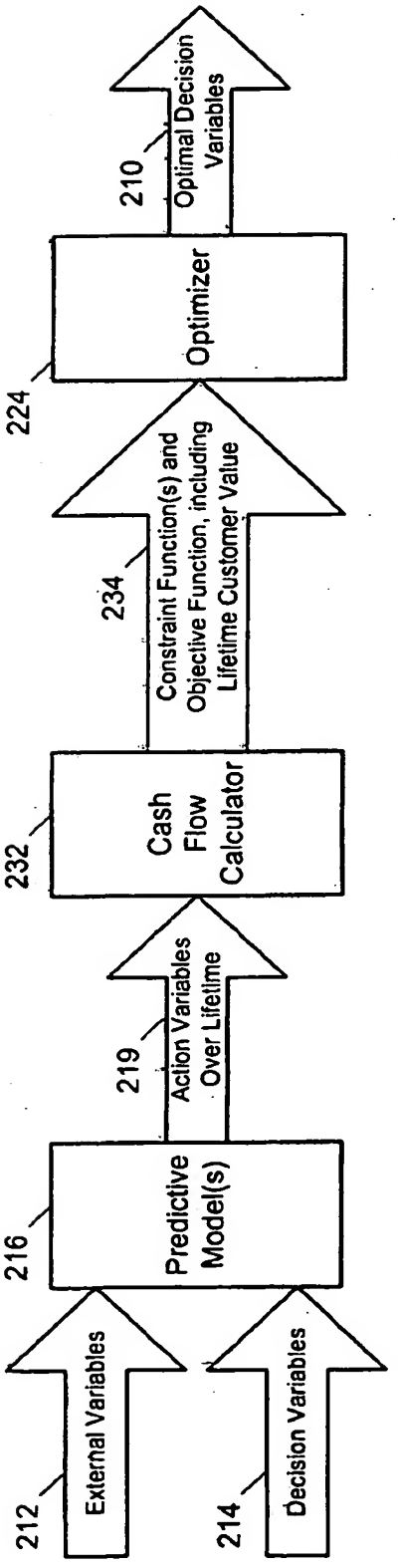

FIG. 11b

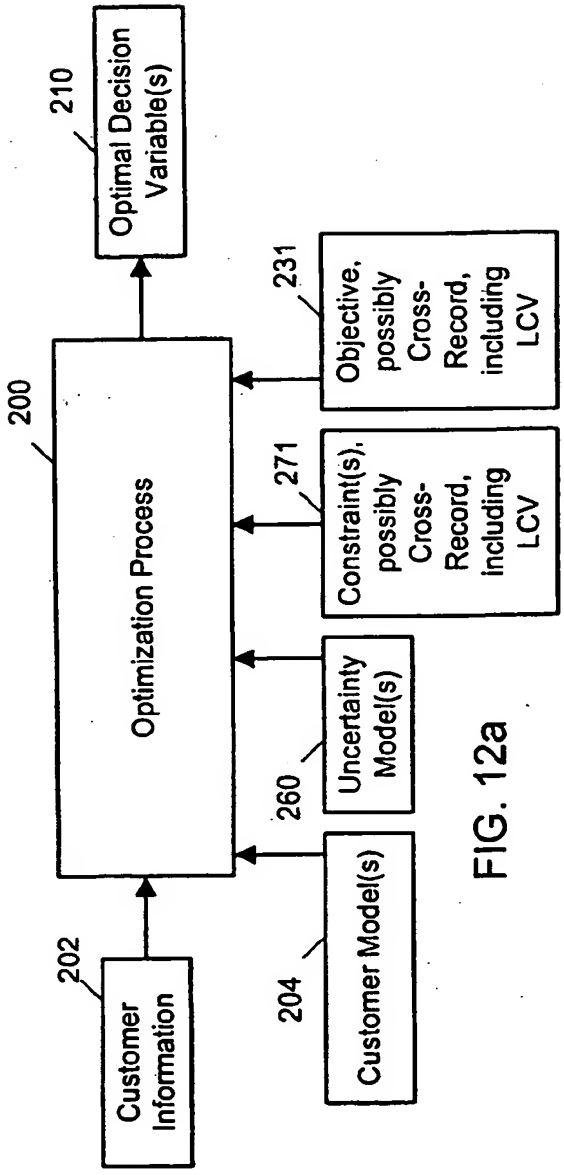

FIG. 12a

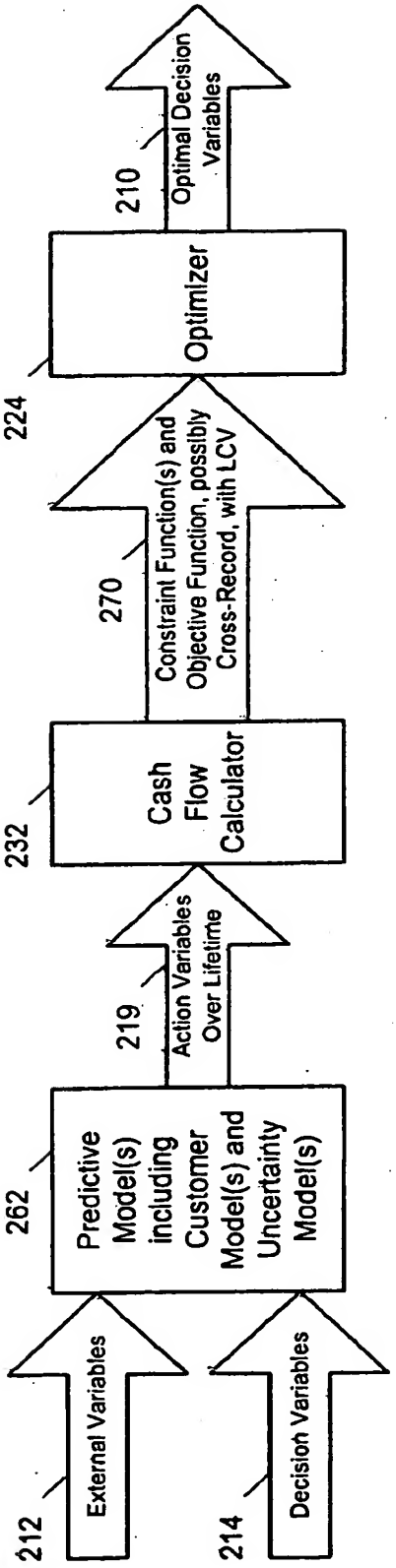

FIG. 12b

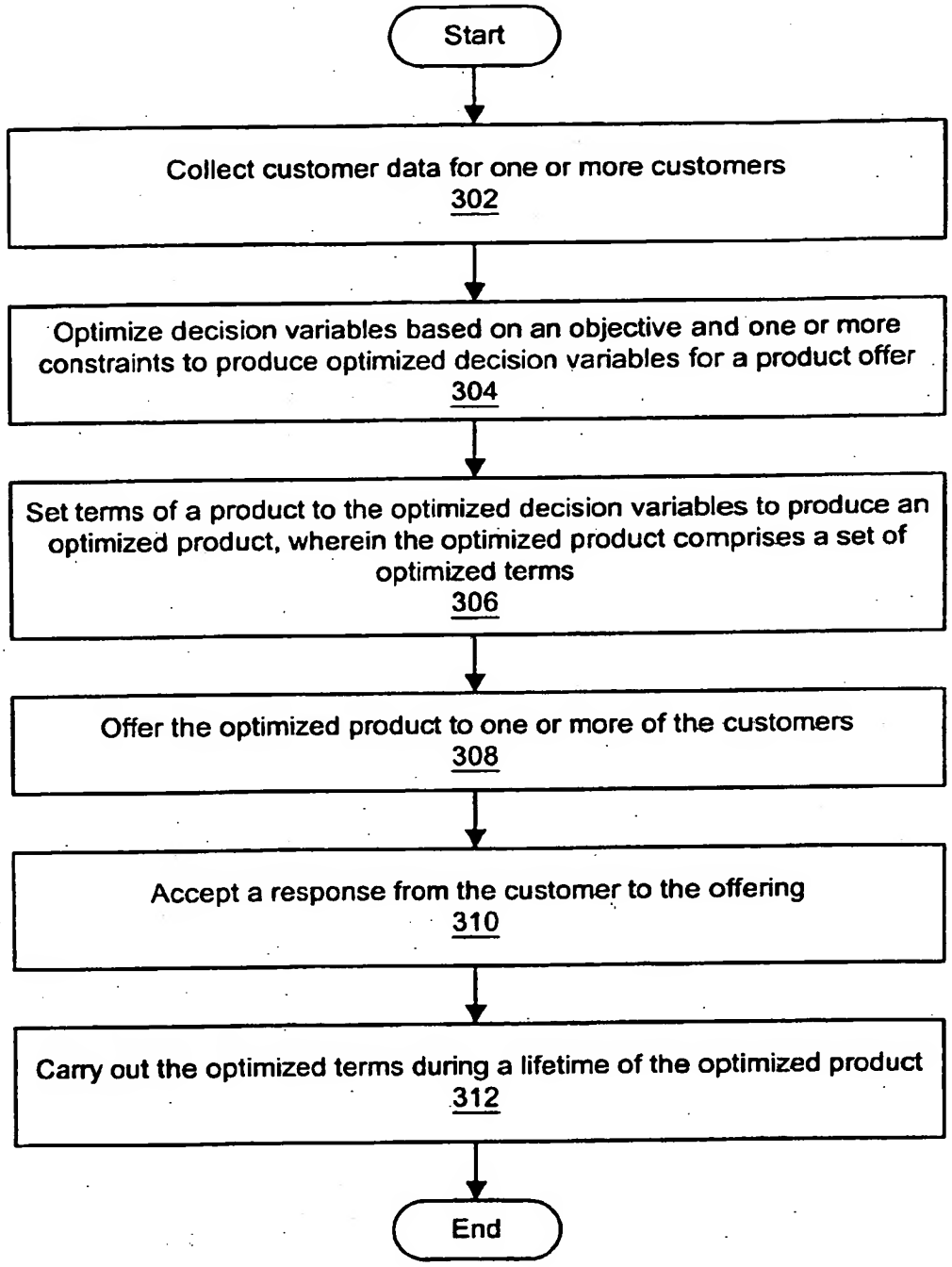
Start
Collect customer data for one or more customers
302
Optimize decision variables based on an objective and one or more constraints to produce optimized decision variables for a product offer
304
Set terms of a product to the optimized decision variables to produce an optimized product, wherein the optimized product comprises a set of optimized terms
306
Offer the optimized product to one or more of the customers
308
Accept a response from the customer to the offering
310
Carry out the optimized terms during a lifetime of the optimized product
312
End

FIG. 13

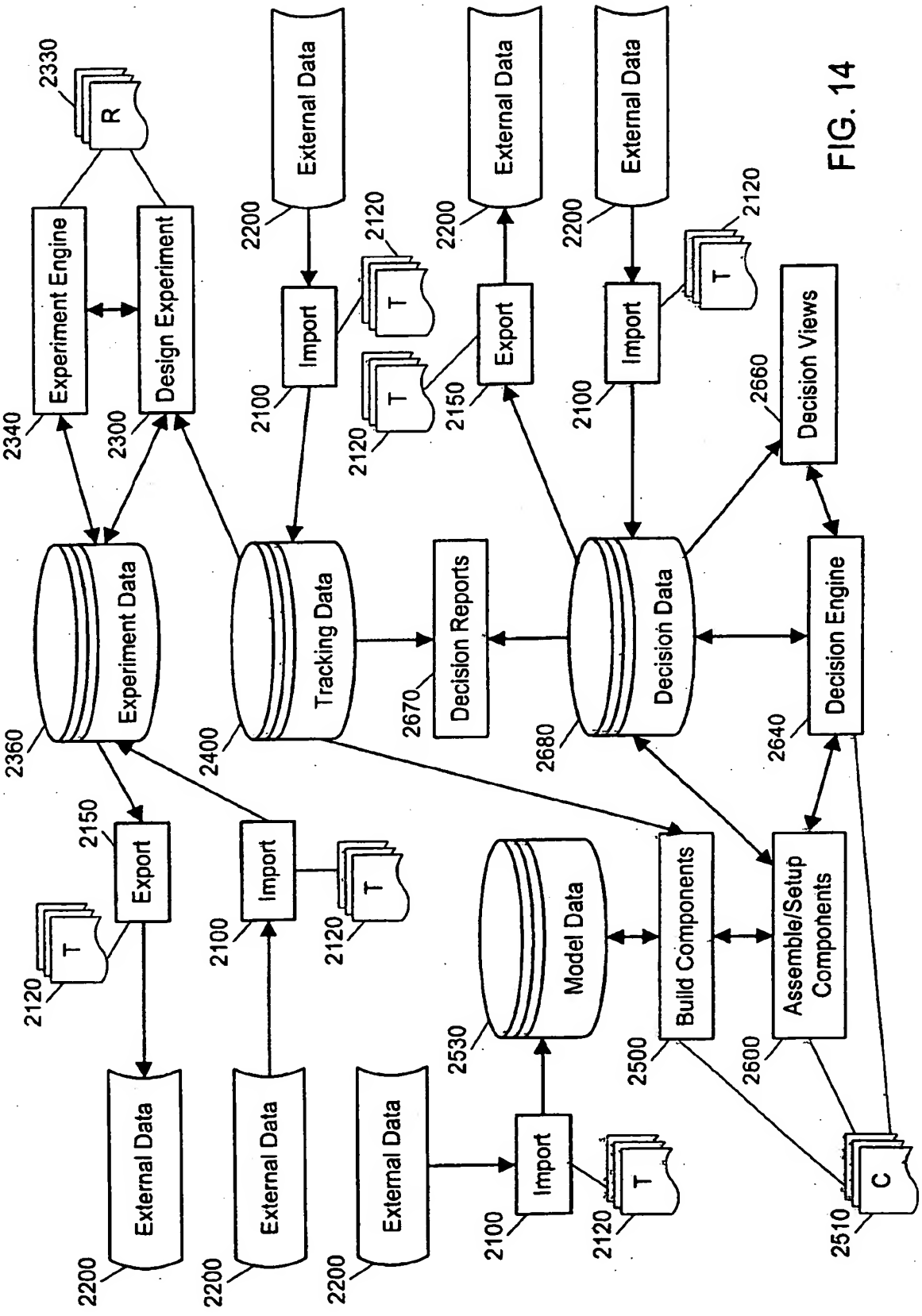
2330
R
Experiment Engine
Design Experiment
2340
2300
External Data
2200
Import
2100
T
2120
Export
2150
Experiment Data
2360
Tracking Data
2400
Decision Reports
2670
Decision Data
2680
Decision Views
2660
Decision Engine
2640
Model Data
2530
Build Components
2500
Assemble/Setup Components
2600
C
2510

FIG. 14

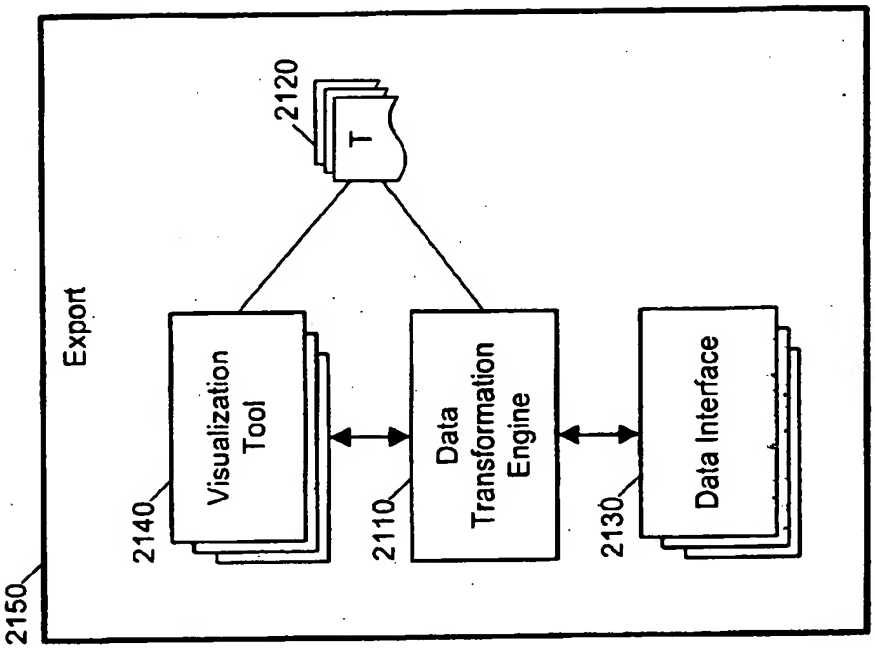
Export
2150
2120
T
2140
Visualization Tool
2110
Data Transformation Engine
2130
Data Interface

FIG. 16

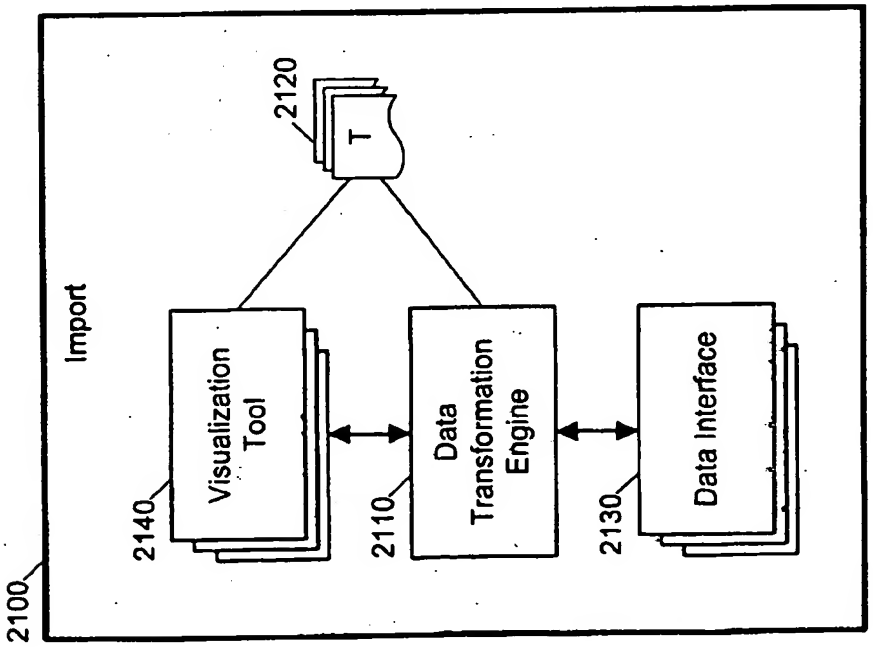
Import
2100
2120
T
2140
Visualization Tool
2110
Data Transformation Engine
2130
Data Interface

FIG. 15

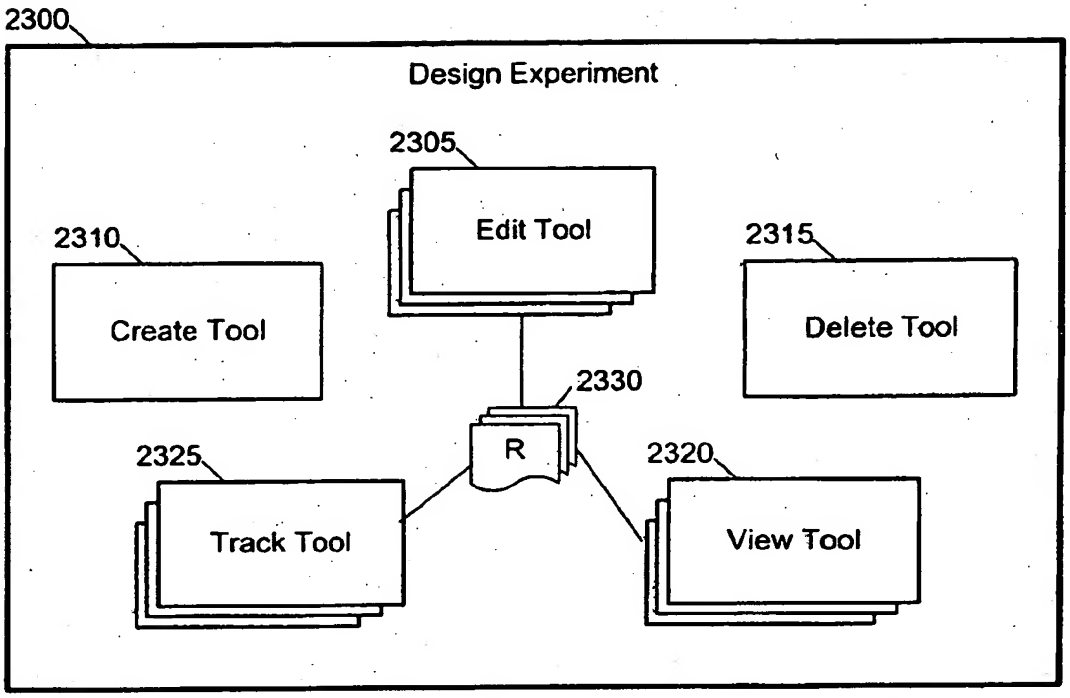
2300
Design Experiment
2305
Edit Tool
2310
Create Tool
2315
Delete Tool
2330
R
2325
Track Tool
2320
View Tool

FIG. 17

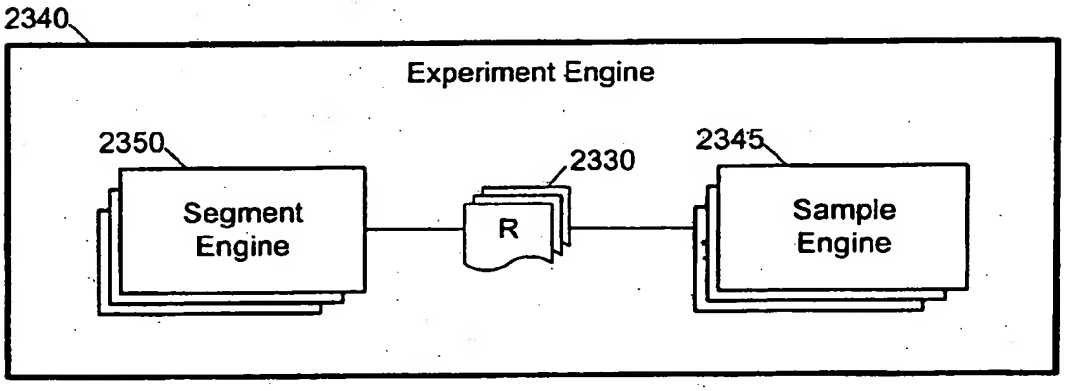
2340
Experiment Engine
2350
Segment Engine
2330
R
2345
Sample Engine

FIG. 18

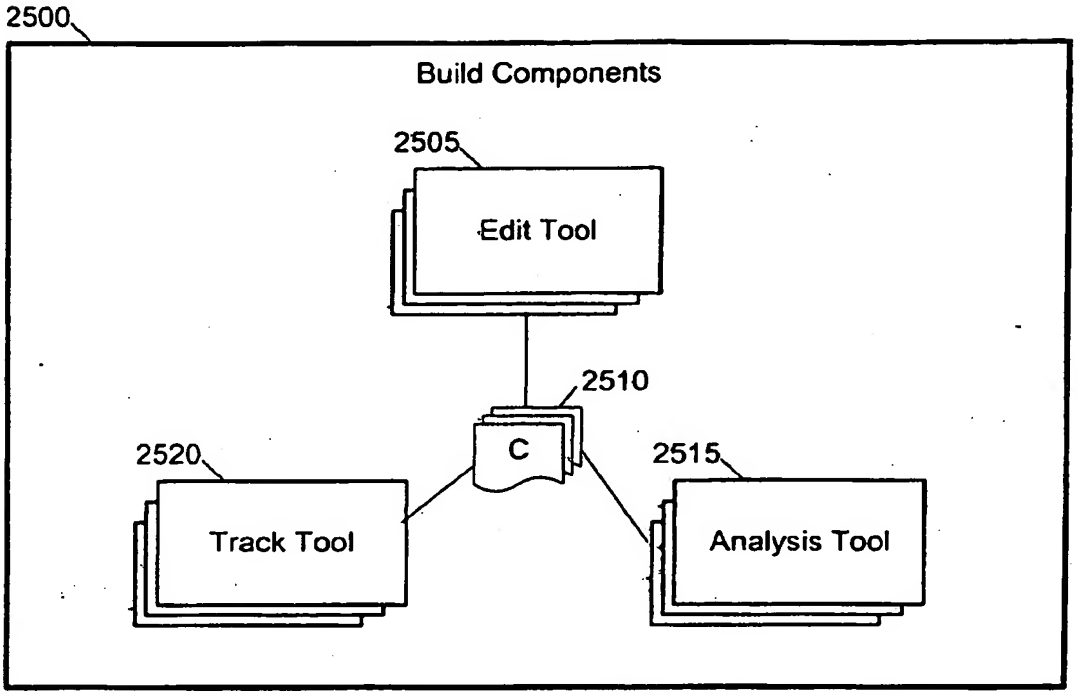
2500
Build Components
2505
Edit Tool
2510
C
2520
Track Tool
2515
Analysis Tool

FIG. 19

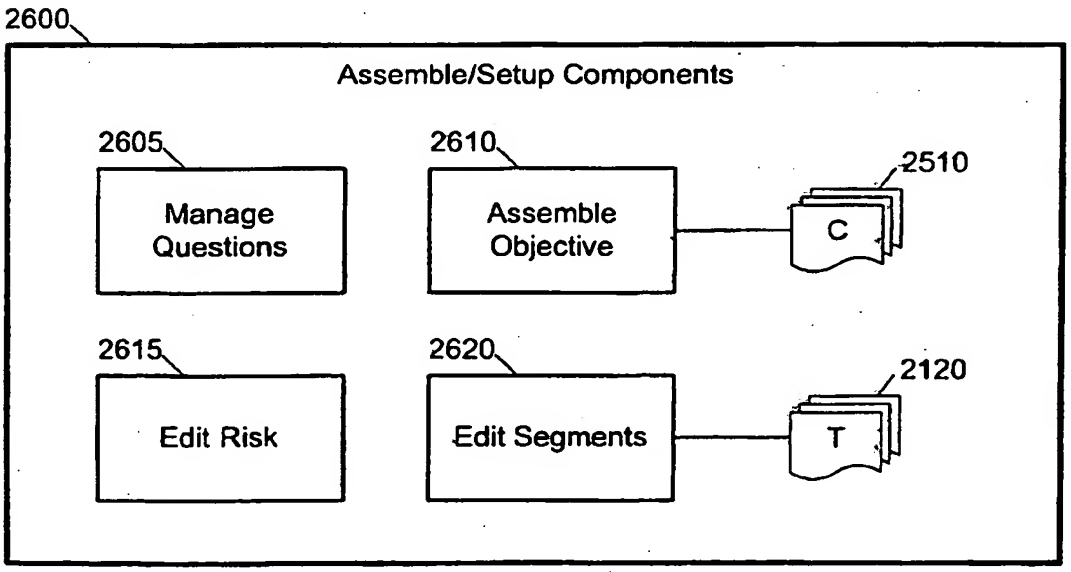
2600
Assemble/Setup Components
2605
Manage Questions
2610
Assemble Objective
2510
C
2615
Edit Risk
2620
Edit Segments
2120
T

FIG. 20

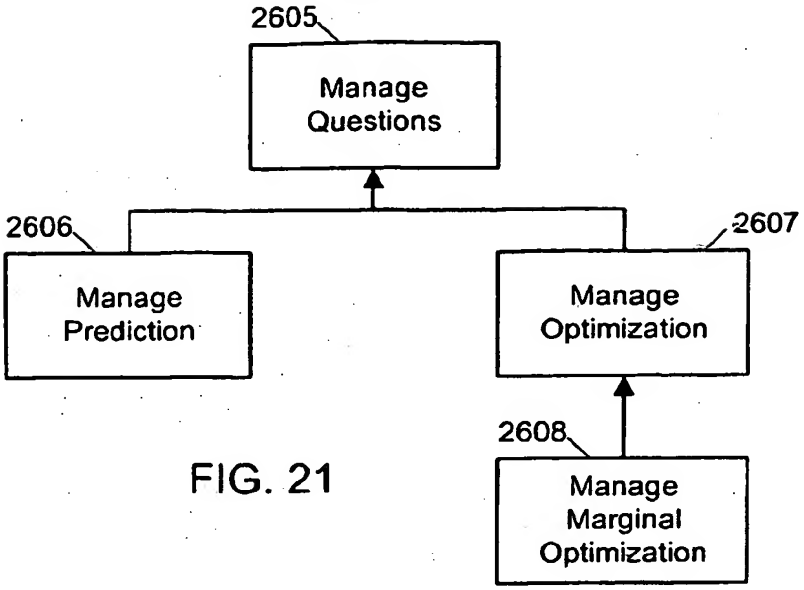

FIG. 21

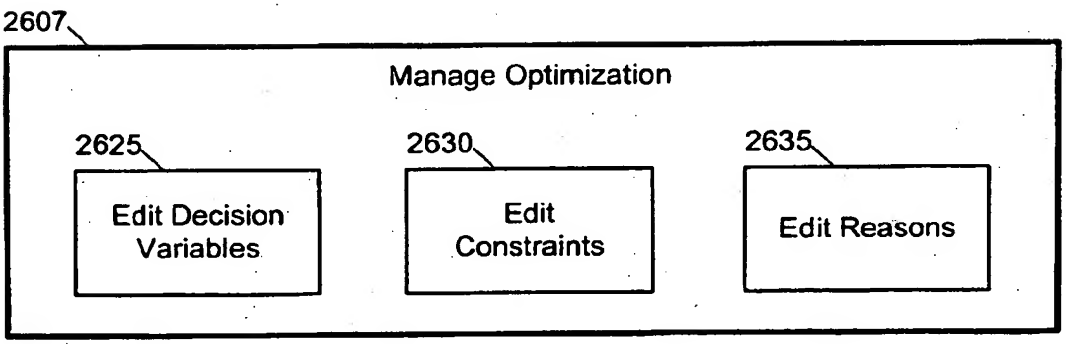

FIG. 22

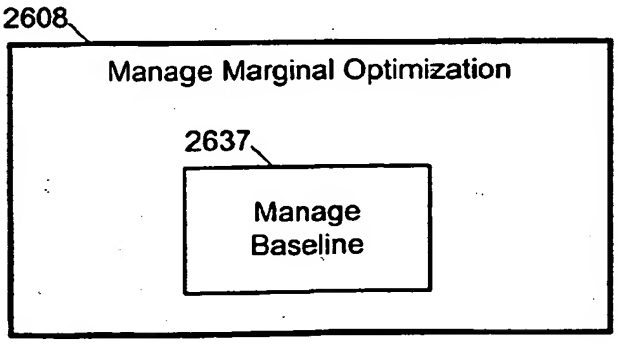

FIG. 23

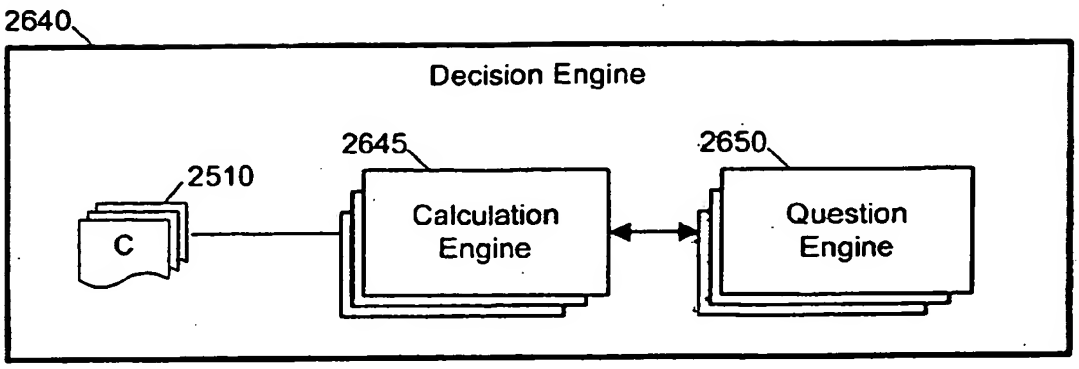

FIG. 24

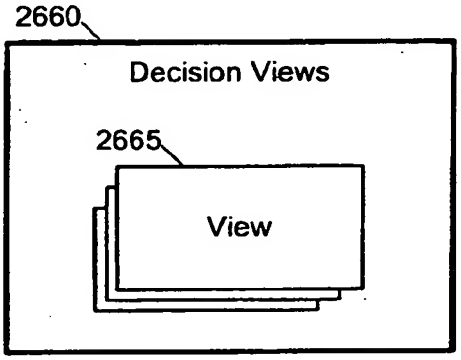

FIG. 25

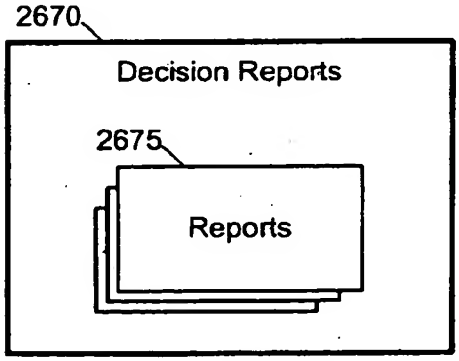

FIG. 26

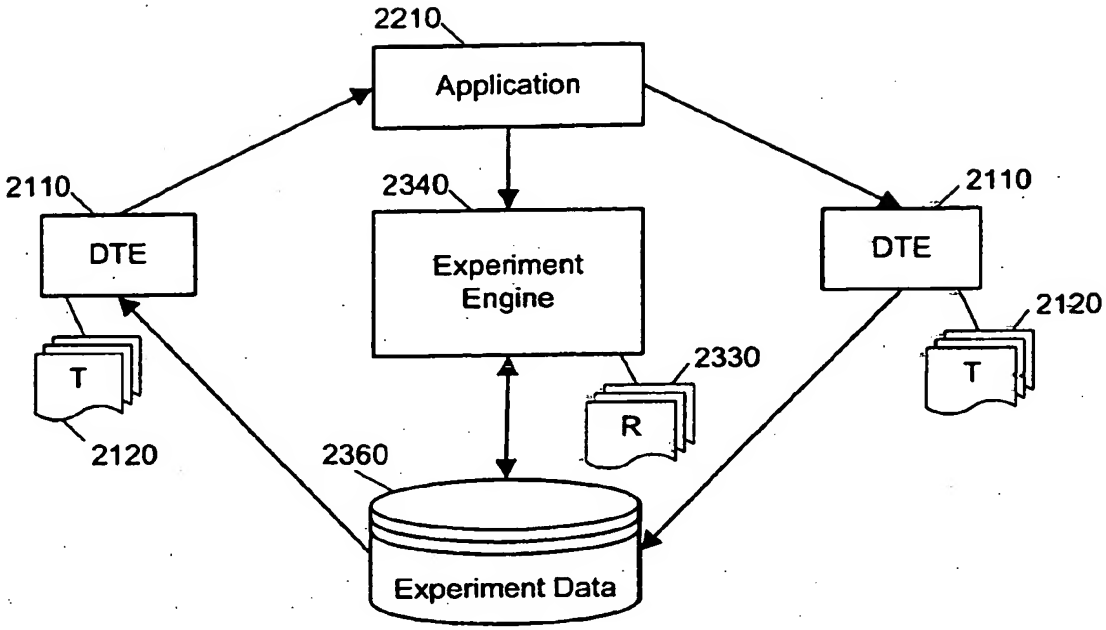
2210
Application
2110
DTE
2340
Experiment Engine
2110
DTE
2120
T
2330
R
T
2120
2360
Experiment Data

FIG. 27

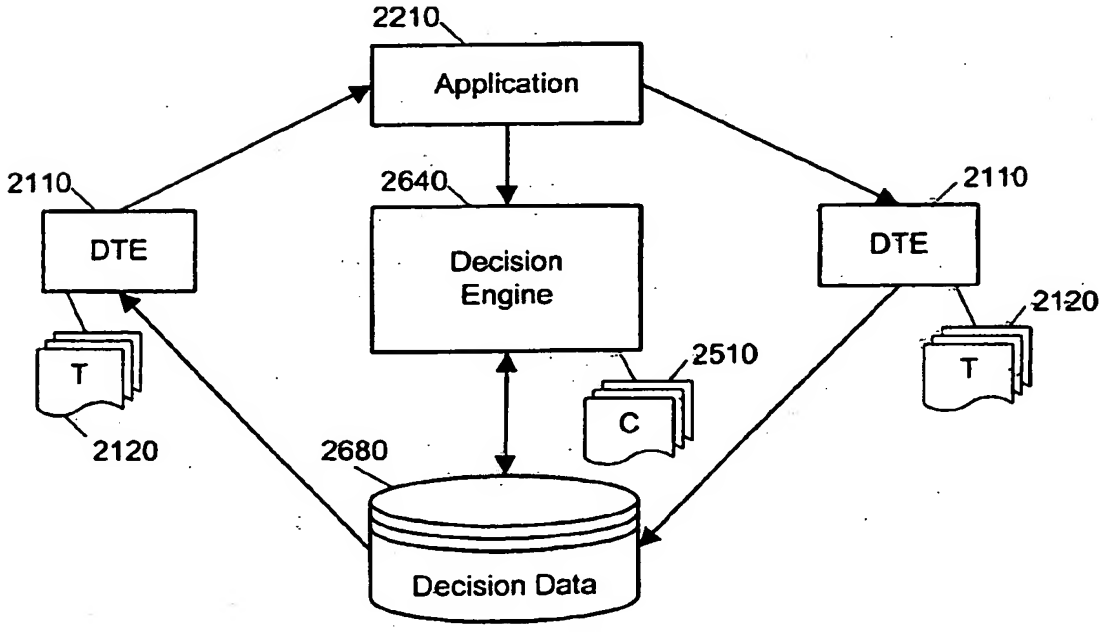
2210
Application
2110
DTE
2640
Decision Engine
2110
DTE
2120
T
2510
C
T
2120
2680
Decision Data

FIG. 28

PATENT COOPERATION TREATY

PCT

REC'D 21 MAY 2001
WIPO PCT

DECLARATION OF NON-ESTABLISHMENT OF INTERNATIONAL SEARCH REPORT

(PCT Article 17(2)(a), Rules 13ter.1(c) and Rule 39)

Applicant's or agent's file reference	IMPORTANT DECLARATION	Date of mailing *(day/month/year)*
5652-00401		19/04/2001
International application No.	International filing date *(day/month/year)*	(Earliest) Priority date *(day/month/year)*
PCT/US 01/ 02880	26/01/2001	28/01/2000
International Patent Classification (IPC) or both national classification and IPC	G06F17/60	
Applicant		
TRAJECTA, INC.		

This International Searching Authority hereby declares, according to Article 17(2)(a), that **no international search report will be established** on the international application for the reasons indicated below

1. ☐ The subject matter of the international application relates to:
   - a. ☐ scientific theories.
   - b. ☐ mathematical theories
   - c. ☐ plant varieties.
   - d. ☐ animal varieties.
   - e. ☐ essentially biological processes for the production of plants and animals, other than microbiological processes and the products of such processes.
   - f. ☐ schemes, rules or methods of doing business.
   - g. ☐ schemes, rules or methods of performing purely mental acts.
   - h. ☐ schemes, rules or methods of playing games.
   - i. ☐ methods for treatment of the human body by surgery or therapy.
   - j. ☐ methods for treatment of the animal body by surgery or therapy.
   - k. ☐ diagnostic methods practised on the human or animal body.
   - l. ☐ mere presentations of information.
   - m. ☐ computer programs for which this International Searching Authority is not equipped to search prior art.

2. ☐ The failure of the following parts of the international application to comply with prescribed requirements prevents a meaningful search from being carried out:
   ☐ the description ☒ the claims ☐ the drawings

3. ☐ The failure of the nucleotide and/or amino acid sequence listing to comply with the standard provided for in Annex C of the Administrative Instructions prevents a meaningful search from being carried out:
   - ☐ the written form has not been furnished or does not comply with the standard.
   - ☐ the computer readable form has not been furnished or does not comply with the standard.

4. Further comments: SEE FURTHER INFO

Name and mailing address of the International Searching Authority	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL-2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Lucia Van Pinxteren

FURTHER INFORMATION CONTINUED FROM PCT/ISA/ 203

A meaningful search is not possible on the basis of all claims because all claims are directed to - Scheme, rules and method for doing business - Rule 39.1(iii) PCT

The applicant's attention is drawn to the fact that claims relating to inventions in respect of which no international search report has been established need not be the subject of an international preliminary examination (Rule 66.1(e) PCT). The applicant is advised that the EPO policy when acting as an International Preliminary Examining Authority is normally not to carry out a preliminary examination on matter which has not been searched. This is the case irrespective of whether or not the claims are amended following receipt of the search report or during any Chapter II procedure. If the application proceeds into the regional phase before the EPO, the applicant is reminded that a search may be carried out during examination before the EPO (see EPO Guideline C-VI, 8.5), should the problems which led to the Article 17(2) declaration be overcome.